Whitepaper

i-Ready Fluency Flight

Doing Math Fact Fluency Right

How Fluency Flight Combines the Latest Research on Learning, Neuroscience, and Psychology with the Best Practices in Digital Games to Build Students' Fluency

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"My students don't know their basic math facts."

"My students have memorized some facts, but they don't understand them."

"Time pressure makes some of my students nervous—they're starting to dislike math."

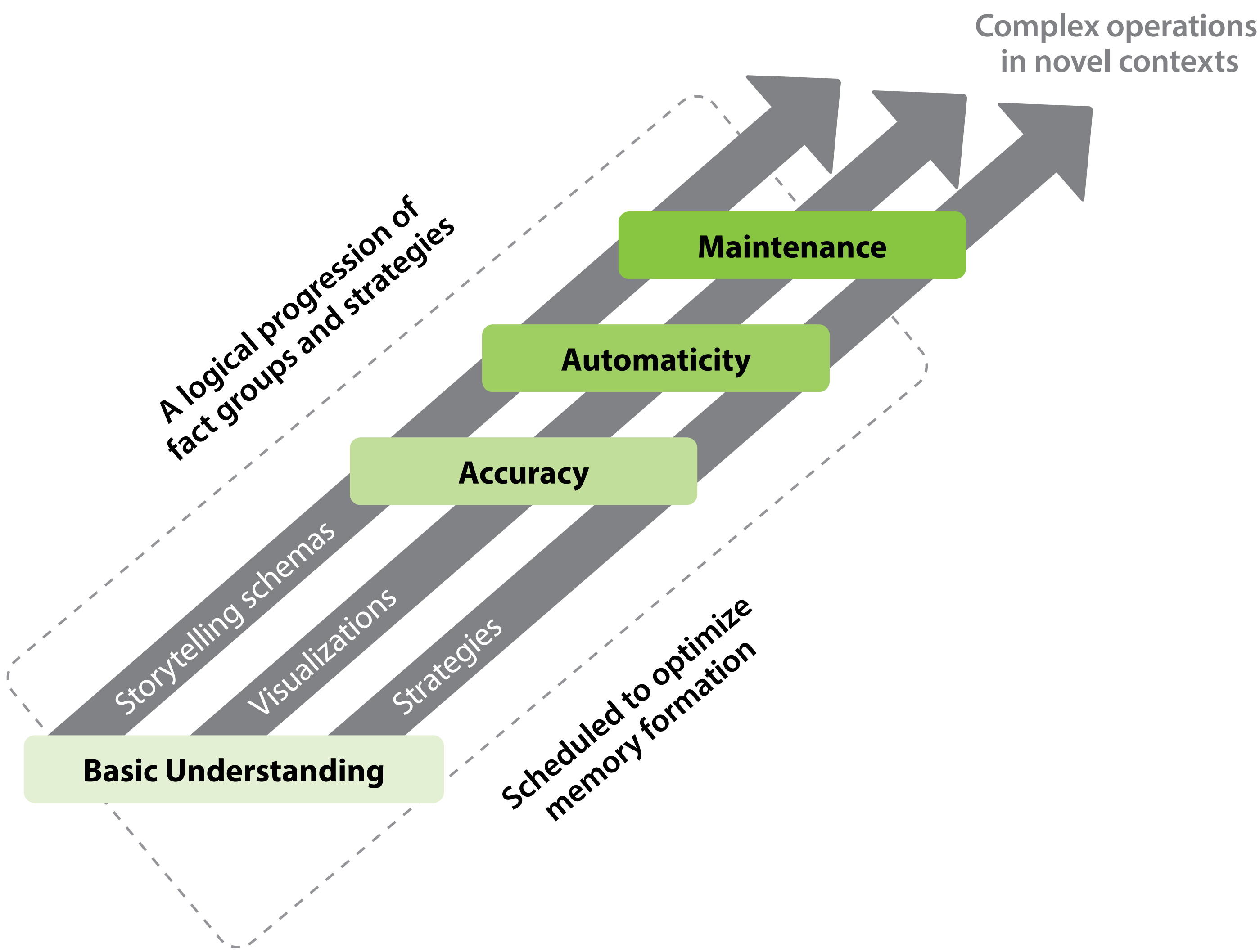
"My students need more practice."

Automaticity and Understanding, without Anxiety

To address the far-too-common educator concerns listed on the previous page, we've created Fluency Flight, a digital solution designed to help students achieve math fact automaticity—with understanding and without speed anxiety. This document provides an overview of the pedagogy and research underlying our approach.

Fluency Flight supports students in chronological Grades 2–5, helping them build math fact fluency in addition and subtraction facts within 20 and multiplication and division facts with operands to 12. Fluency Flight's digital activities and games cover facts and strategies, such as Doubles or Make a Ten, that progress through the four stages of fluency: basic understanding, accuracy (i.e., procedural reliability), automaticity (i.e., recall), and maintenance.

Our goal is not just to help students achieve automaticity of essential math facts, but to do so through understanding concepts and internalizing efficient strategies. In addition to the pure, symbolic practice that's found in most fact fluency solutions, Fluency Flight provides storytelling schemas, visualizations, and strategy building. With this foundational triad, students will not only achieve basic fact automaticity, but they'll also understand the meaning of operations, the relationships between facts, and powerful strategies that can be applied to more complex operations in novel contexts.



Theoretical Foundations

Our approach merges three traditions in the domain of fact fluency:

1. Build Conceptual Understanding with Procedural Fluency

The first is the modern consensus, promoted by the National Research Council (National Research Council, 2001) and the National Council of Teachers of Mathematics (Gojak, 2012), that fact fluency should be developed in conjunction with conceptual understanding and in the context of procedural fluency. Kanter and Leinwand (2018), and especially Bay-Williams and Kling (2019), provide a framework for this approach, which calls for developing fluency in stages: Students must first form a basic understanding of the operations, then learn foundational facts in a pedagogically sound progression, and finally, learn to apply derived fact strategies appropriately, accurately, efficiently, and flexibly. In this system, timed evaluations are strongly discouraged until students master strategies, particularly because early introduction of time pressure can lead to anxiety and encourage students to rely on quick but non-extensible counting methods rather than to commit to the initially slower path of strategy building. Bay-Williams and Kling (2019) cite a range of studies demonstrating that strategy building is not only an effective path to achieving fact fluency but also a mechanism for increased math achievement overall (Vasilyeva et al., 2015).

> Fact fluency should be developed in conjunction with *conceptual understanding* and in the context of *procedural fluency*

2. Support Long-Term Memory Retention and Retrieval

The second is a wealth of research in the areas of neuroscience, psychology, and the behavioral sciences that reveals best practices for maximizing long-term memory retention and retrieval for the purpose of achieving fluency. These include strategic spacing, interleaving, and interspersing of content, as well as alignment of content modalities with stages of fluency development. Studies have shown that spacing at the level of seconds and even minutes prevents repetition suppression (Xue et al., 2011), while spacing at the level of hours (e.g., one to four) increases long-term potentiation of neural structures related to memory (Kramár et al., 2012). The next critical intervals occur every 24 hours for the next three days, during which nearly 100% of nascent memories may be retained if reviewed on a daily interval, but otherwise may be lost (Ebbinghaus, 1885; Schwartz et al., 2011). Once memories are consolidated, long-term retention is maximized by spaced repetition of recall, with the retention interval increasing in proportion to the spacing interval (Walsh et al., 2018). Additional studies have shown that interleaving unrelated content during learning can create a beneficial spacing effect (Codding et al., 2019), while interleaving related content can boost inductive category learning and transfer (Carvalho & Goldstone, 2014). Studies of math fluency interventions have further demonstrated that increasing the ratio of unlearned to learned content during encoding, while decreasing the same ratio during retention activities, can have a positive effect on long-term learning (Axtell et al., 2009). Similarly, new learning is enhanced by the presence of fewer practice problems with immediate feedback (Gersten et al., 2009), while retention practice benefits from a high volume of problems followed by more summative reviews (Davis et al., 2017).

> Best practices for maximizing long-term memory retention and retrieval for fluency include:
>
> 1. Strategic spacing
> 2. Interleaving
> 3. Interspersing of content
> 4. Alignment of content modalities with stages of fluency development

Our approach provides fluency content on a schedule that aligns with these principles of memory formation to enhance learning, retention, and automatic recall. See Appendix A for a description of how these principles are applied in the solution.

3. Engage and Motivate with Effective Digital Games

The third is domain knowledge of what works well in a digital medium, specifically on student devices with minimal teacher intervention, including, but not limited to these principles:

a. Game mechanics can be used not only to entertain, but to educate; students should be empowered to playfully interact with math visualizations to better understand the structure of numbers and operations.

b. Digital activities should have interfaces that can be learned quickly with intuitive physical controls and interactions and concise tutorials so students can focus on the math content.

c. Moments of delight should reward student effort and persistence.

d. Success should always feel more interesting than failure.

Our Approach

Approach to Engagement

Fluency building requires consistent and repeated sessions of learning and practice (Cozad & Riccomini, 2016). To engage students in this process, we provide:

1. Simple, clear communication of usage expectations and actual usage. In Fluency Flight, there is an infographic in the student interface that displays the target and completed number of days per week.
2. A variety of different activities, ranging from low-barrier exploration to high-performance, pure practice.
3. A structured sequence of content with a selective amount of choice. During each session, students are allowed to choose between a range of activities (e.g., practice versus new learning) once they complete a compulsory activity.
4. A virtual landscape that represents overall student progress and engages them with a mix of short-term and long-term goals to unlock and explore. This metaphor conveys growth and even decay, the latter highlighting the need for skills maintenance.

Approach to Content

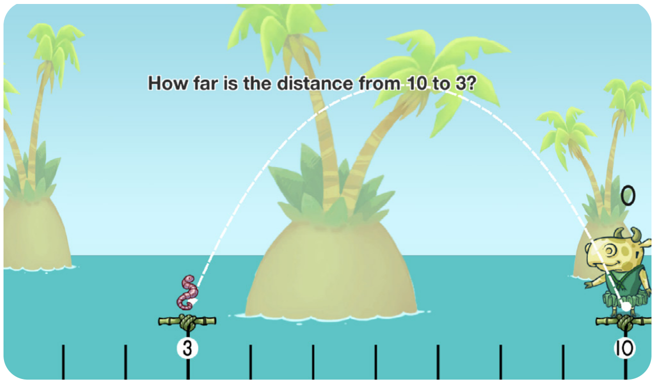
Content follows a logical progression, adapted from Bay-Williams and Kling (2019), in which foundational operations and facts form the basis for learning more complex operations, facts, and strategies. At a high level:

1. Addition/Subtraction Foundational Facts (e.g., Add within 5, Use Partners for 10)
2. Addition/Subtraction Derived Fact Strategies (e.g., Make a 10, Use Near Doubles)
3. Multiplication/Division Foundational Facts (e.g., 2s, 10s, 5s, 1s, 0s, Squares)
4. Multiplication/Division Derived Fact Strategies (e.g., Add a Group, Break Apart)

Within each of the four operations, facts are presented in a progression that is distinct from the traditional—and still prevalent—sequencing of fact memorization, which precedes in an ascending fashion from zeroes to 10s. Although orderly in appearance, this approach doesn't leverage the relationships between facts in ways that build deeper mathematical understanding and expedite learning. By contrast, students commence with the most conceptually accessible facts (e.g., 2s, 5s, and 10s for multiplication) and then learn strategies to derive those facts that remain (e.g., the Subtract a Group strategy facilitates learning 4s and 9s based on the student's knowledge of their 5s and 10s). These strategies are therefore both the vehicles for learning facts and the tools that can be utilized for more advanced contexts and operations to come.

For each set of foundational facts or derived fact strategies, students progress through content tailored to facilitate four distinct stages of learning outlined on the next page. These stages generally align with several extant progressions, including instructional hierarchy (Haring et al., 1978), instructional levels (Burns et al., 2006), and hypotheses (Daly et al., 1997) concerning low academic performance.

The content available at each stage is optimized to maximize growth in terms of activity, scaffolds, and scheduling. See Appendix A for more information about the content progression in each stage.



1 **Basic Understanding**—The goals of this stage are to help students:

a. Understand the meaning of operations via storytelling schemas

b. Build mental models of facts using visualizations such as number lines, ten-frames, and groups of dots

c. Learn strategies through demonstrations*

2 **Accuracy**—The goal of this stage is to solve math fact problems accurately (with no time requirement). The content in this stage is more abstract, including symbolic problem presentations.

3 **Automaticity**—The goal of this stage is to solve math fact problems accurately and quickly. The content is similar to that of the Accuracy stage, but speed encouragement is introduced.

4 **Maintenance**—The goal of this stage is to retain automaticity of facts. The content is the same as the Automaticity stage but provided on increasingly spaced intervals.

**Schemas and strategies are introduced with known applicable facts, then applied to new applicable facts, and finally explored with less applicable facts so students build an understanding of when and how they apply. This stage is also critical for establishing the visualizations, language, and concepts that will be used as feedback if students need support in later stages.*

Approach to Feedback

Given that Fluency Flight builds both conceptual understanding and automaticity, we provide rich, targeted feedback with the following characteristics:

1. Aligned with stages of fluency. While students are building basic understanding and working on accuracy, feedback is immediate and formative. Feedback at these stages also leverages visualizations to aid in concept formation. When students are working on automaticity or maintenance of facts and skills, feedback is more summative as to avoid interrupting the flow of recall exercises. Feedback at these stages leans toward pure symbols and offers students the opportunity to repair their understanding with appropriate variations on the classical Taped Response* and Cover–Copy–Compare fluency interventions**.
2. Non-punitive. Rather than focusing solely on errors, feedback celebrates successes and highlights known facts. This both motivates and promotes metacognitive awareness of progress.
3. Multiple representations of feedback are provided in order to bridge connections, make comparisons, and build strong conceptual models.
4. Feedback is responsive. Regardless of the type of activity, if a student needs support, the activity slows down and offers remedial feedback to avoid unnecessary frustration.

Approach to Speed

The importance of rapid recall of facts and application of strategies to overall mathematics education achievement and beyond is well documented (Cozad & Riccomini, 2016), and achieving automaticity in core math facts has even been shown to reduce math anxiety (Wittman, 1995). However, speed pressure, particularly if applied early in the learning process, can inhibit strategy formation, increase math anxiety, and even delay attainment of automaticity (Hasselbring et al., 1987). As such, we approach speed with care.

1. Speed will be encouraged with positive reinforcement, but slow accuracy will not be penalized. We will avoid any sense of time pressure.
2. Speed encouragement is not introduced for a set of facts until the student reaches a sufficient threshold of accuracy. This encourages students to take the requisite amount of time to engage in basic understanding and strategy building.
3. Once speed challenges are introduced, the solution focuses on celebrations of personal progress and achievements, while avoiding social pressures or competitions.

**In a Taped Response intervention, students must attempt to correctly solve problems before an audio recording provides the answers on a given interval (Codding et al., 2011).*

***In a Cover–Copy–Compare intervention, students are first presented with a completed problem. The problem is then covered, and the student is asked to copy the problem from memory. Finally, the student compares their version with the original problem (Codding et al., 2011).*

Our Solution

Recommended Usage

The implementation guidelines for Fluency Flight represent a balance between research-based best practices and practical limitations. At a high level, we recommend:

4 days each week

8–10 minutes each day

1. Four days of Fluency Flight per week, which amounts to roughly eight to 10 minutes per day. This is reduced to one day per week of maintenance practice once a student has achieved automaticity for all facts and strategies.
2. We encourage—but do not require—bonus sessions, particularly at the start of each week. A bonus session provides a review of newly learned material following a break of at least one hour after completing the primary content for the day. This will serve to maximize long-term potentiation of new learning.

Content Progression

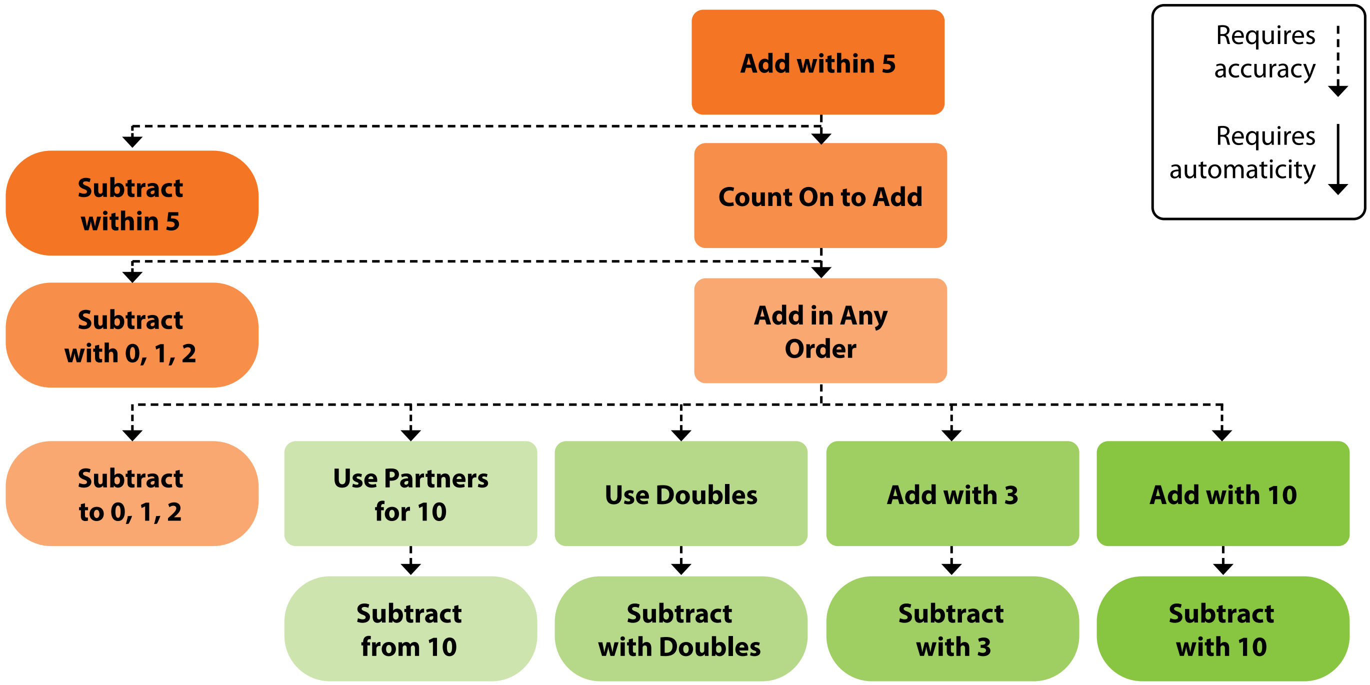
Each step in the progression through addition and subtraction, then multiplication and division, represents a **Fluency Focus**—either a logical grouping of foundational facts or a derived fact strategy. Students advance through **Fluency Stages** for each focus—from working on basic understanding, to working on accuracy, to working on automaticity, to working on maintenance—in accordance with their performance in aligned activities. Dependent foci are unlocked as students achieve accuracy or automaticity for underlying foci.

The progression for addition and subtraction is shown on the next page; priority of unlocked foci proceeds from the top down, from left to right.

Content Progression, Cont'd.

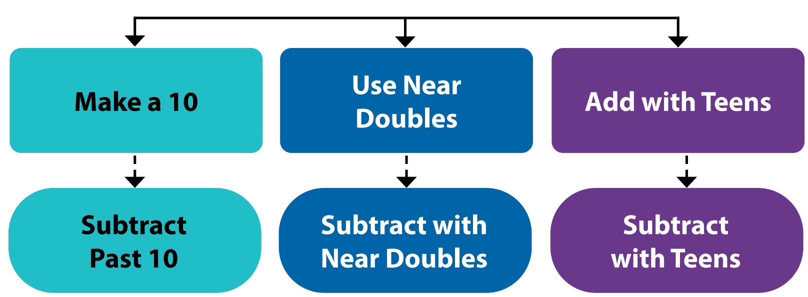
Addition and Subtraction Progression

Foundational facts



Derived Facts

Once automaticity with all foundational facts is achieved



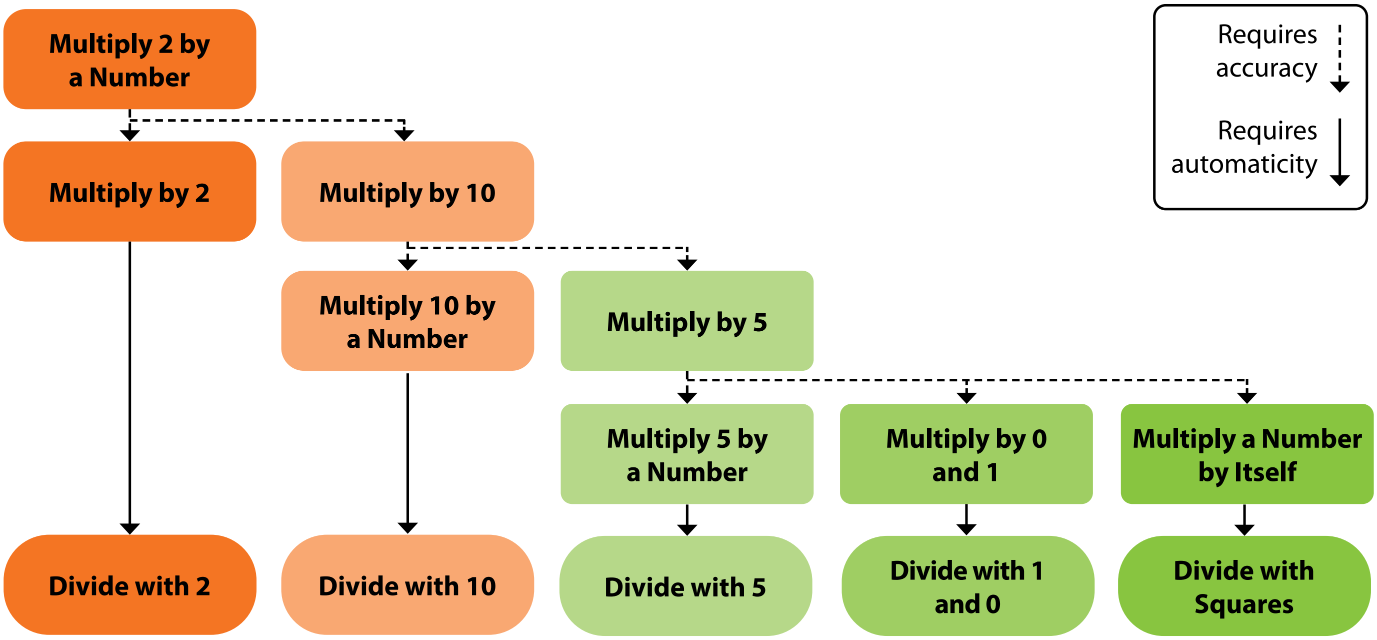
Note:

- The commutative property is introduced early in the progression. After a student is accurate in *Count On to Add* facts (e.g., +0, 1, 2), we introduce the commutativity with the complementary *Add in Any Order* facts (e.g., 0, 1, 2+).
- Subtraction facts are introduced as students achieve accuracy in corresponding addition facts.
- Students must achieve automaticity in all foundational addition and subtraction facts before they can unlock derived fact strategies.

The content sequence and unlock rules for multiplication and division are shown on the next page.

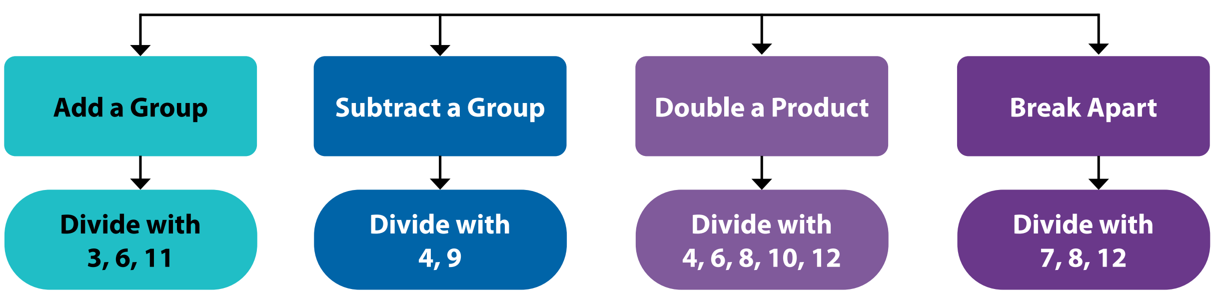
Multiplication and Division Progression

Foundational facts



Derived Facts

Once automaticity with all foundational facts is achieved



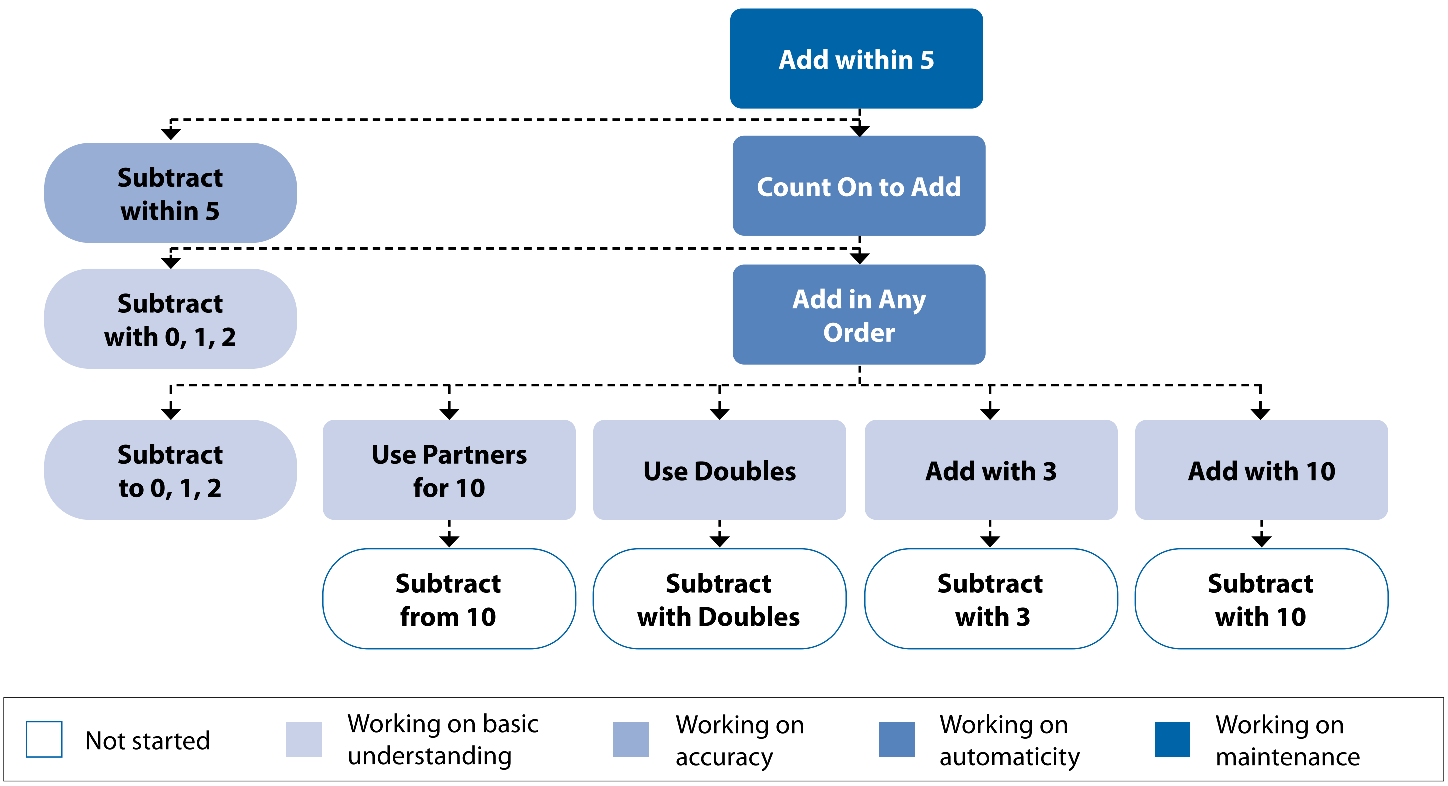
Note:

- As with addition, the commutative property of multiplication is introduced early in the content progression.
- Division of related facts is introduced as students achieve automaticity in corresponding multiplication facts. This is a higher bar than for subtraction because learning subtraction is supported by multiple strategies that complement the learning of addition, while division is mainly achieved with the strategy of "think multiplication," for which automaticity is essential.
- Basic understanding for multiplication is unlocked once a student achieves accuracy with foundational addition/subtraction facts.
- Derived fact strategies are unlocked once a student achieves automaticity in all foundational multiplication and division facts.
- Students in Grade 2 may not proceed past basic understanding in multiplication (i.e., Grade 2 students will not be exposed to explicit multiplication terms or symbolic multiplication activities). Students in other grades have no limit on how far they can progress through the content.

Content Progression, Cont'd.

- The prioritization of content across operations is as follows:
  - Accuracy in addition/subtraction
  - Accuracy in multiplication/division
  - Automaticity in addition/subtraction
  - Automaticity in multiplication/division

At any point in their overall fluency progression, a student will be at various stages in one or more foci. The following graphic shows the state of a hypothetical Grade 2 student after a month of use. This student has already achieved automaticity on sums within 5, and those facts have entered a long-interval maintenance cycle. At the same time, the student is working on automaticity in sums involving 0, 1, and 2 and is working on accuracy in subtraction. Finally, the student has recently unlocked and is working on basic understanding in subtraction facts involving 0, 1, and 2 as well as the remainder of their foundational addition facts.

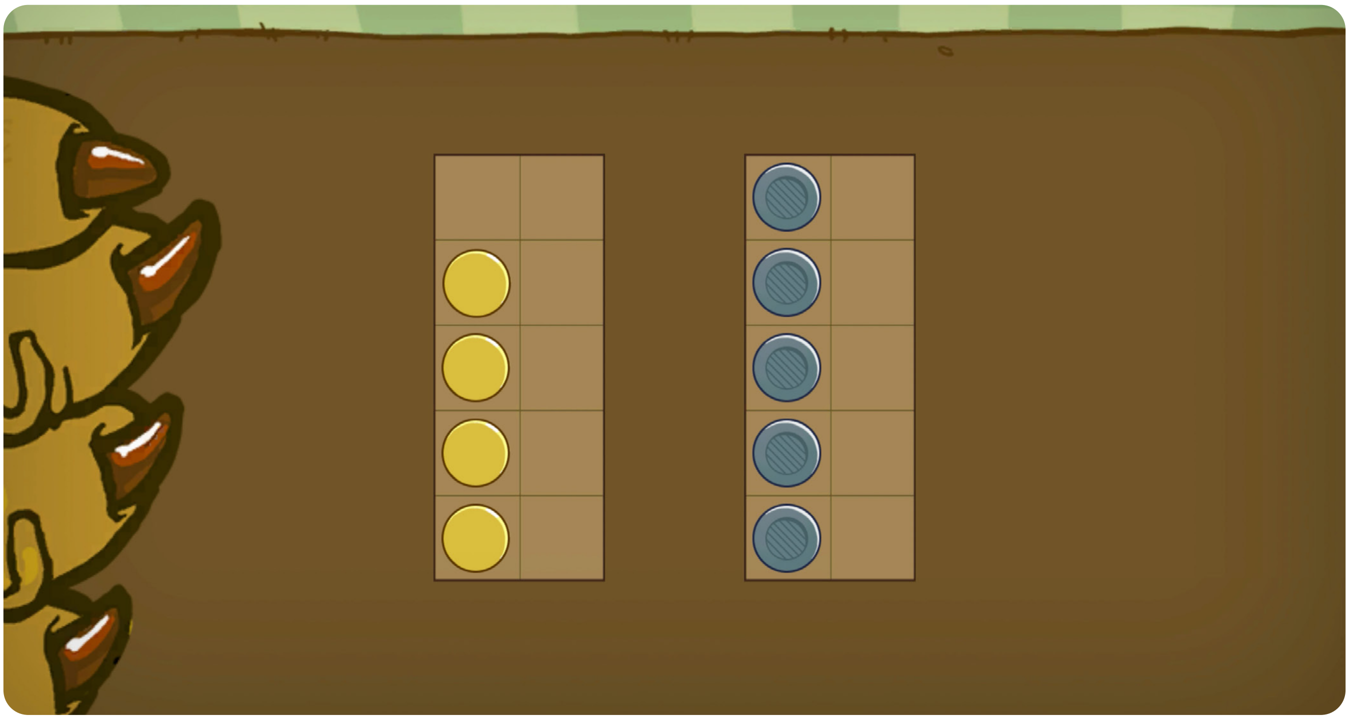


Content Recommendations

At the start of each day, Fluency Flight generates a set of recommended activities based on the student's active fluency foci and stages and their recent activities. The recommendation process yields a mix of compulsory and elective content designed to maximize memory consolidation, retention, and variety.

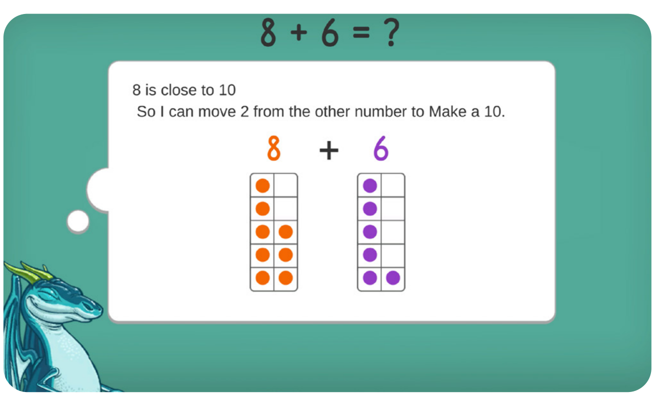
Activities

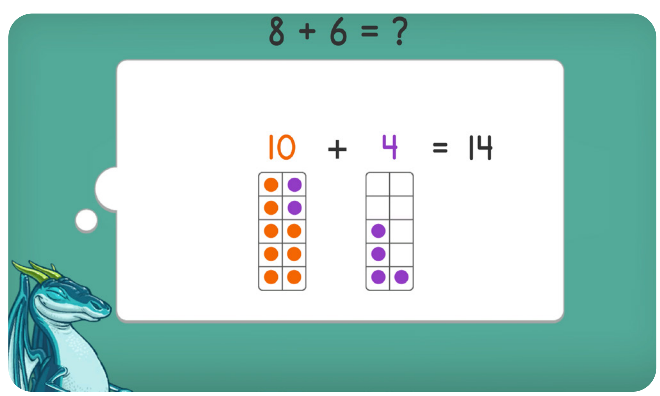
Fluency activities are aligned with particular fluency foci and stages in terms of content, learning goals, and affordances. Activities also support a degree of self-pacing—students performing well will progress through an activity more quickly, while students who need support will proceed at a slower pace with more feedback. Sneak Peek—Near Doubles is an example of a quick-look visualization for a student working on **basic understanding** of the Use Near Doubles strategy. The sneak peek format encourages students to subsidize, scan for patterns, and derive strategies rather than count for the answer. In this case, the student is presented with two ten-frames that represent the problem 4 + 5 and is asked to enter the total. (Note the monster's hand, which covers the problem after a few seconds to discourage counting.)



Sneak Peek—Near Doubles is an example of a quick-look visualization for a student working on basic understanding of the Use Near Doubles strategy.

Make a Ten is an example of an interactive strategy introduction for a student working on **basic understanding** for the Make a Ten addition strategy. In the example below, the student is presented with the problem 8 + 6, with both addends shown as dots in ten-frames. An animation demonstrates how moving two dots from the 6 to the 10 transforms the problem into the more easily solved 10 + 4. After a few visual and symbolic examples, students solve their own related problems, with visual scaffolds.





Make a Ten is an example of an interactive strategy introduction for a student working on basic understanding for the Make a Ten addition strategy.

Activities, Cont'd.

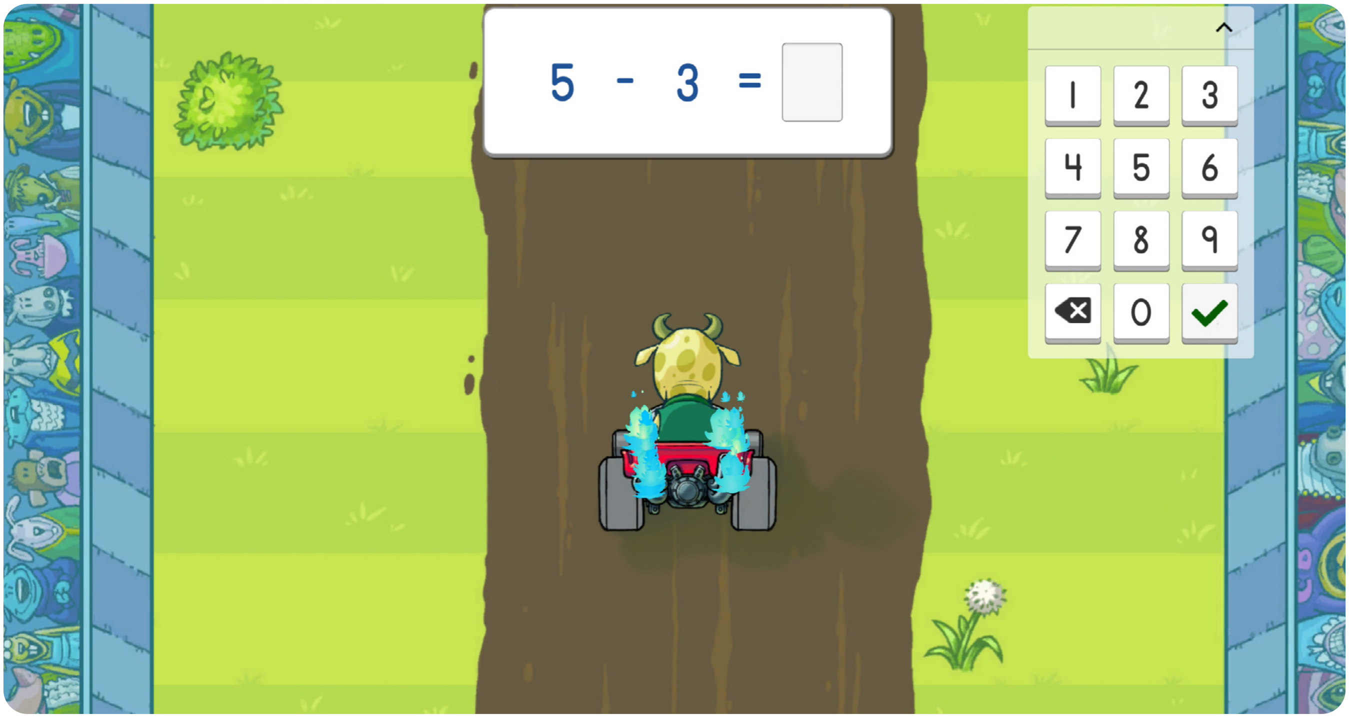
Food Frenzy is an example of a **basic understanding** activity to help students in the early stage of making sense of multiplication. Students first create an array by clicking or dragging—in the example shown below, six rows of two bottles of ketchup. After creating the array, the student is asked to select the correct product.

Food Frenzy is an example of a basic understanding activity to help students in the early stage of making sense of multiplication.

Bird Brunch is an example of an activity for a student working on **accuracy** for all subtraction facts. A number line from 0 to 10 stretches across the screen with a player's avatar standing on a platform. The image below shows the problem 10 – 3. The player must enter 7 to jump the correct distance to reach the next bird snack. Note that this activity can also be used for a student working on automaticity by adding a speed bonus and changing the feedback format for incorrect answers from formative to summative.

Bird Brunch is an example of an activity for a student working on accuracy for all subtraction facts.

Fact Racer is an example of an automaticity activity for all addition, subtraction, multiplication, and division facts. Students enter answers into an open text box, which powers their race car. Faster entry speeds the car up even more. Midway through the race, there's a pit stop to review correct and incorrect answers.

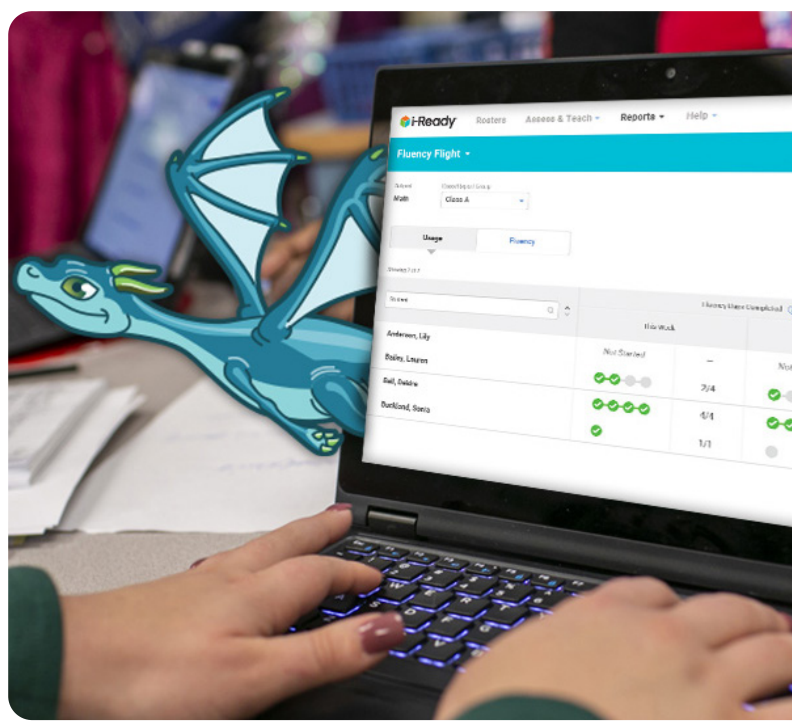


Fact Racer is an example of an automaticity activity for all addition, subtraction, multiplication, and division facts.

Teacher Reports

Our solution provides teachers with key insights into their students' progress with their math facts, including:

1. Usage data with a calendar of the previous four weeks to help encourage consistent implementation
2. Progress toward students' accuracy and automaticity with addition, subtraction, multiplication, and division facts
3. *Coming soon!* Accuracy and automaticity scores for each math fact for each student
4. Eventually, Fluency Flight will include reporting on the fluency stage each student is working on for each fluency focus.



Appendix A: Characteristics of the Four Stages of Fluency

	Working on Basic Understanding	Working on Accuracy	Working on Automaticity	Working on Maintenance
Goals	Contextualize, visualize, and make sense of operations, facts, and strategies	Practice new learnings until fact problems are answered accurately	Practice new learnings until fact problems are answered accurately and quickly	Retain automaticity of facts
Activities	Storytelling schemas Visualizations Strategy demonstrations	Games/practice with symbolic problems	Games/practice with symbolic problems and speed encouragement	(Same as automaticity)
Instructional Tools	Demonstrations Guided Practice Immediate Feedback	Formative feedback (including strategies and visualizations) for incorrect answers	Summative feedback (including strategies and visualizations) for incorrect answers	
Problem Interleaving[1]	Blocked practice	Interspersed	Interspersed	
Unlearned to Learned[2]	100:0	80:20	20:80	
Opportunities to Respond[3]	Low frequency	Medium frequency	High frequency	
Inter-Problem Interval[4]	Long	Medium	Short	
Measures	Basic understanding is demonstrated via completing a small number and type of activities. Students who need additional support will not be in basic understanding for long as they will continue to see strategy and visualization as feedback in later stages.	Accuracy is measured at the fact level (for both fact sets and strategies) and aggregated at the foci. Threshold: 80%–95% accurate	Automaticity is measured at the fact level (for both fact sets and strategies) and aggregated at the foci. Threshold: ≤ 3 seconds	
Memory/ Scheduling	Encoding/Consolidation Days: 1, 1.5, 2, 3, 4 *Repeat until basic understanding is achieved.*	Encoding/ Consolidation Days: 1, 1.5, 2, 3, 4 *Repeat until accurate.*	Encoding/ Consolidation Days: 1, 1.5, 2, 3, 4 *Repeat until automatic.*	Retention Days: 8, 16, 32, . . . *Continue with maximum interval of two months.*

[1]The degree to which the same or similar problems are interspersed with different content. Blocked practice indicates repeated practice of the same or similar problems. Interspersed content alternates between problems of different types.
[2]Increasing the ratio of unlearned to learned content during encoding, while decreasing the same ratio during retention activities, can have a positive effect on long-term learning.
[3]Opportunities to Respond is a term for the number of events that elicit a student response. Passive demonstrations or long inter-trial intervals result in fewer opportunities to respond (versus rapidly presented sequences of problems).
[4]An adaptation of the clinical "inter-trial interval," which is essentially the time between presentation of problems.

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