



Elevate Lessons in Mathematics Research Base

Mathematics



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INTRODUCTION

A growing body of research shows there is a strong foundation of practices in mathematics instruction that, when applied to curriculum, supports all students in reaching proficiency.

Curriculum Associates develops its curriculum based on research about how students learn and the skills they need to succeed academically. Elevate Lessons support near- and on-grade level learners—middle schoolers who have unfinished learning from a prior grade but who don't need support with elementary grade numeracy skills. Essential Lessons support older striving learners in Grades 6–8 who are two or more grade levels behind. The research base for Essential Lessons is available [here](#).

Based on the results of their beginning-of-year *i-Ready* adaptive assessment, students are placed in either Essential Lessons or Elevate Lessons. Both provide supplemental mathematics instruction to be used alongside daily teacher-led lessons in their core curriculum.

The following pages outline the research base upon which Elevate Lessons were built and how it delivers explicit, systematic instruction in foundational mathematical concepts to near- and on-grade level learners.

Overview of How Elevate Lessons Align with the Research

Elevate Lessons comprehensively address all critical domains specified by the National Council of Teachers of Mathematics (NCTM) and the Common Core State Standards (CCSS)—Number and Operations, Algebra and Algebraic Thinking, Measurement and Data, and Geometry—while aligning with key recommendations from research.

- The *i-Ready* adaptive assessment places each student in a personalized learning path through online lessons, ensuring instruction builds on prior knowledge and provides an appropriate level of challenge.
- Elevate Lessons balance guided instruction with independent practice that requires students to interpret problems, analyze given information, and determine how to proceed.
- Elevate Lessons support students to construct meaning around mathematical concepts, properties, and procedures.
- Elevate Lessons begin with guided questions and/or an exploratory activity that introduces a mathematical situation related to the lesson topic without explicit explanation. This approach helps students make connections to their prior knowledge in a low-stakes environment that fosters a growth mindset through perseverance.

- Problem-solving activities in Elevate Lessons involve representing real-world situations using mathematical models. Students use tools to model the problem at hand, and if they struggle, the system provides scaffolded support.
- Problem scenarios used in Elevate Lessons are designed to be meaningful and relevant to students while being culturally and linguistically responsive.

ELEVATE LESSONS OVERVIEW

Elevate Lessons, the latest addition of *i-Ready Pro* for middle school, provide near- and on-grade level learners with efficient, focused, and evidence-based instruction aligned with grade-level mathematics. Elevate Lessons ensure a seamless learning experience for both near- and on-grade level learners by providing targeted support for foundational mathematical concepts while staying connected to their grade-level curriculum.

Students begin with the *i-Ready* adaptive assessment, which assesses a broad range of mathematics skills across four domains: Number and Operations, Algebra and Algebraic Thinking, Measurement and Data, and Geometry. This assessment helps determine each student's readiness and instructional needs in each of these domains.

Students who are one grade level below in the Number and Operations domain are placed into Elevate Lessons. These near-grade level learners first engage with Prerequisite Lessons to reinforce key concepts from the previous grade before transitioning to grade-level content. On-grade level lessons provide near- and on-grade level learners with personalized just-in-time support to scaffold their learning and address misunderstandings. Some lessons also offer an optional 10-minute prerequisite refresher. The lessons provide support and adaptivity for striving students while organizing lessons into manageable topics. Elevate Lessons' modular design allows for flexible, targeted learning experiences and helps teachers align instruction with their core curriculum.

THE EVIDENCE BASE

Importance of Grade-Level Content for Near- and On-Grade Level Learners

Ensuring that all students, including those with unfinished learning from prior grades, receive instruction aligned with grade-level skills is essential for educational belonging. Research from The New Teacher Project (TNTP)'s *The Opportunity Myth* reveals that many students, particularly those from underserved communities, are often denied access to grade-level instruction despite having the potential to succeed (TNTP, 2018). Studies show that when students engage with grade level-appropriate assignments, they make significantly greater academic progress than those who do not (TNTP, 2018). Additionally, research on co-requisite remediation in community colleges suggests that students can successfully engage with advanced material while receiving targeted support on prerequisite skills, reinforcing the importance of structured learning acceleration (Center on Reinventing Public Education, 2022).

Effectively implementing grade-level instruction requires a strong differentiation strategy that provides just-in-time support without slowing students' progress. Research underscores the importance of timely data collection to assess students' understanding and inform targeted interventions (Center on Reinventing Public Education, 2022). Just-in-time prerequisite reviews have been shown to be effective in addressing learning gaps while maintaining access to grade-level content (NCTM & NCSM, 2020). Aligned instruction creates a seamless support system, enabling students to reinforce prerequisite knowledge while engaging in rigorous coursework (Tntp, 2021). Additionally, continuous performance monitoring and real-time feedback are crucial in helping students succeed in challenging academic environments (Rollins, 2014).

While traditional remediation often focuses too heavily on reteaching past material, leaving students perpetually behind, simply focusing on grade-level instruction without addressing these gaps isn't the solution either. As New Classrooms notes, "focusing solely on grade-level standards precludes students from having the opportunity to address their unfinished learning from prior years" (New Classrooms Innovation Partners, 2019). Because mathematics is cumulative, missing key foundational concepts can prevent students from succeeding with more advanced material.

Supporting Grade-Level Success for Near- and On-Grade Level Learners

Elevate Lessons provide grade-level content with embedded just-in-time prerequisite supports, ensuring students can engage with rigorous material while receiving the scaffolding they need for academic success. Elevate Lessons leverage data from *i-Ready* adaptive assessments to provide each student with their own personalized pathway that directly supports grade-level instruction. Elevate Lessons deliver differentiated support and ongoing assessment, adjusting each student's experience based on demonstrated needs. This approach ensures that near-grade level students receive targeted instruction to bridge learning gaps while staying engaged with grade-level coursework.

Research on Growth Mindset

A substantial body of research shows that fostering a growth mindset—by encouraging persistence and effort—is more effective than reinforcing the belief that intelligence and ability are fixed (Dweck, 2007).

Strong Research Support for a Growth Mindset

According to the researcher Carol Dweck, students who believe their intelligence is fixed are less motivated to apply themselves in school. They become more focused on appearing to be smart rather than developing their abilities, viewing challenges and effort as threats rather than opportunities for growth. When teachers and parents praise innate intelligence, they reinforce students' beliefs that their abilities are fixed, discouraging students from striving to improve (Dweck, 2007).

Multiple studies have examined how students' beliefs about their own intelligence impact their learning (Blackwell et al., 2007; Robins & Pals, 2002; Stipek & Gralinski, 1996). One study found that students who embraced an incremental view of intelligence had higher grades in their first year of junior high school (Blackwell et al., 2007).

Studies by Dweck found that the students who were encouraged to succeed through increased effort and persistence after repeated failure came to see their failure as being due to insufficient effort rather than innate ability. By viewing their intelligence as malleable, they recognized that perseverance leads to success in tasks where they previously needed support (Dweck, 2007).

Two studies tracking 373 junior high school students over a two-year period investigated how students' mindsets influence their learning in mathematics. At the start of the first study, students' mindsets were assessed, and the researchers found that students with a growth mindset tended to experience an upward trajectory in grades, while students who believed their intelligence was fixed tended toward a flat trajectory. The second study designed an intervention to teach students how to develop a growth mindset. The results indicated that teaching students to have a growth mindset (referred to as an "incremental theory") resulted in improved motivation and a reversal in their downward trajectory of grades. Students in the control group were less motivated and continued on a downward trajectory in grades (Blackwell et al., 2007).

As noted by Blackwell et al. (2007), students begin forming fixed or growth mindsets in elementary school, though issues often emerge later when academic work becomes more challenging. Because elementary school is more failure-proof, students with a fixed mindset are less apt to experience struggle until they reach middle school.

Research-Based Recommendations for Mathematics Instruction

Dweck advises that teachers and instructional tools should focus student praise on the steps they take to master learning material rather than on innate intelligence. She explains: "Praise is very valuable . . . if it is carefully worded. Praise for the specific process a child used to accomplish something fosters motivation and confidence by focusing children on the actions that lead to success. Such process praise may involve commending effort, strategies, focus, persistence in the face of difficulty, and willingness to take on challenges" (Dweck, 2007).

Elevate Lessons Foster a Growth Mindset

Elevate Lessons begin with guided questions and/or an exploratory activity. These activities build students' understanding by connecting new concepts to prior knowledge in a low-stakes environment that fosters a growth mindset through productive struggle and perseverance.

For example, in the Multiply and Divide to Solve Inequalities lesson, students first predict whether multiplying both sides of an inequality by the same number always results in a true inequality. Next, they test their prediction by multiplying inequalities by both positive and negative integers. After exploring, they revisit their original hypothesis and practice multiplying both sides of an inequality by the same number to solidify their understanding.

i-Ready Multiply and Divide to Solve Inequalities — Instruction — Level 6

Do you think you can ALWAYS multiply both sides of an inequality by the same number and get a true inequality?

Yes No Not sure

i-Ready Multiply and Divide to Solve Inequalities — Instruction — Level 6

Use the slider to multiply both sides of the inequality by the same value. Do you always get a true inequality? Explore until you are ready to make a conclusion.

$2 > 1$ \rightarrow $2 \times 1 > 1 \times 1$ \rightarrow $2 > 1$
TRUE TRUE

i-Ready Multiply and Divide to Solve Inequalities — Instruction — Level 6

When you multiply both sides of an inequality by the same value, do you ALWAYS get a true inequality?

Yes No Still not sure

i-Ready Multiply and Divide to Solve Inequalities — Instruction — Level 6

You found that when you multiply both sides of an inequality by the same number, you do not always get a true inequality.

Multiply both sides of the inequality by -2 .

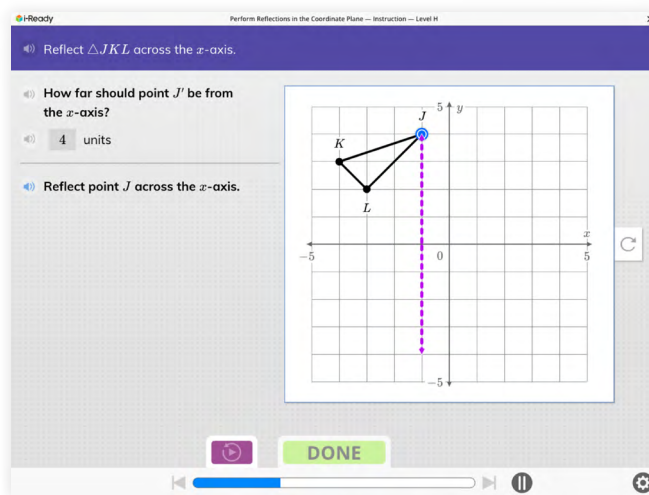
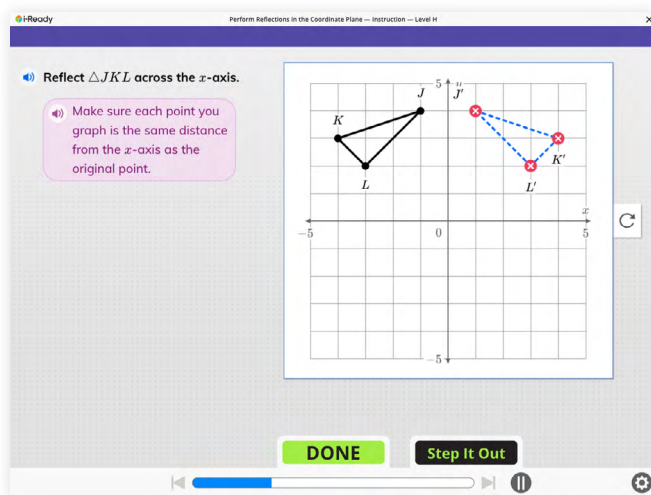
$$4 < 8$$

$$4(-2) < 8(-2)$$

?
<
?

As needed, they receive scaffolded feedback, which may include reminders, tips, help locating essential information, strategy suggestions, or encouragement to try again. When appropriate, students can choose their level of support—another attempt, a stepped-out version of the same problem, or the answer.

In the following example from the Perform Reflections in the Coordinate Plane lesson, students who answer incorrectly receive feedback and can choose to try again or view a stepped-out version of the problem. In the Step It Out, they first determine how far the points should be from the x-axis, then plot them with additional support as needed.



Standards-Based Mathematics Instruction

Two sets of curriculum standards have played a major role in shaping reform in mathematics instruction across the United States: NCTM's Principles and Standards for School Mathematics (NCTM, 2000) and the CCSS for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2023). Most states' curriculum standards have been informed by one or both works, including states that did not formally adopt the CCSS and those that revised their standards after initial adoptions.

In 2014, NCTM published *Principles to Actions: Ensuring Mathematical Success for All*, which recommends actions for mathematics educators, school and district administrators, other education leaders, and policymakers based on the CCSS and more than a decade of new research about excellent mathematics programs and classroom experience. Together, these and related documents provide research-informed guidance on both the "what" and "how" of high-quality mathematics instruction.

The NCTM and CCSS initiatives have engaged in an iterative process of developing and refining best practices in mathematics education. NCTM's Principles and Standards for School Mathematics (2000) was a foundational source for the authors of the CCSS for Mathematics (2010). More recently, NCTM's *Principles to Actions* (2014) provided further research-based guidance for implementing the CCSS effectively.

Their research and documentation share key recommendations for standards-based mathematics instruction:

- Instruction should promote sensemaking and conceptual understanding through meaningful problem solving.
- Students should be encouraged to reason about relationships among mathematical concepts and procedures, recognizing patterns and structure to view mathematics as a system of interrelated ideas.
- Instruction should support students' interpretation and development of simplified models of real-world problems, using multiple representations—graphical, symbolic, and verbal.
- Instructional activities should foster perseverance and productive struggle, helping students engage with challenging mathematics problems.

Aligning with NCTM and CCSS for Effective Mathematics Instruction

Elevate Lessons align with NCTM and CCSS by addressing all critical mathematical domains and incorporating key research-backed recommendations. The lessons emphasize meaning making in mathematics, integrating conceptual, factual, and procedural knowledge to build a strong foundation for learners. It provides problem-solving contexts that encourage students to model mathematical challenges while fostering persistence. Through targeted instruction and practice, Elevate Lessons help students develop key skills for content across domains—Number and Operations, Algebra and Algebraic Thinking, Measurement and Data, and Geometry—ensuring a comprehensive and effective approach to mathematics learning.

Research on Meaning Making in Mathematics

There is broad agreement among researchers, mathematicians, and the NCTM Standards about the value of having students construct meaning in the context of mathematical problem solving (Burns, 2015; Lesh & Zawojewski, 2007; Hiebert, 2003; NCTM, 2000; Sfard, 2003).

Research and expert opinions consistently support the idea that meaning making is essential for successful mathematics learning. Helping students understand the structural relationships between mathematical concepts is key to this process. There is consensus among researchers that students can better construct meaning in the context of mathematical problem solving. Successful mathematics instruction has shifted toward supporting more meaningful learning by building on students' prior knowledge, providing opportunities for intervention and practice, exploring multiple problem-solving methods, and encouraging students to justify their reasoning. While skills development remains important, it should be incorporated into students' construction of knowledge.

Learning as Meaning Making

Meaning making is the ability for a learner to understand their surroundings and the world around them as a way to motivate and ground their learning. As learners grow, they develop the ability to construct meaning, which ultimately leads to their successfully grasping content (Sfard, 2003). Structure is essential for developing understanding and making meaning.

Prominent theorists have weighed in on this important relationship between seeing structural relationships and developing understanding. Sfard (2003) cites Skemp's (1976) distinction between knowing rules without reasons and knowing both what to do and why. In mathematics, knowing why involves understanding the structural relationships among concepts, properties, and procedures, emphasizing connecting mathematical structures to real-world problem scenarios as a way to foster deeper understanding.

Comparing Traditional and Alternative Approaches to Meaning Making

Sfard (2003) highlights instructional shifts recommended by NCTM's Principles and Standards for School Mathematics, which move away from teacher-driven instruction, rote memorization, and mechanistic answer finding. Instead, the focus shifts toward reasoning, problem solving, and building meaningful connections between mathematical concepts. Similarly, Hiebert's (2003) research contrasts traditional instruction, which focuses on basic procedural understanding, with alternative approaches that build on students' prior knowledge, encourage problem solving, and ask students to compare methods and justify their solutions.

Comparing Impacts of Traditional and Meaning-Based Mathematics Instruction

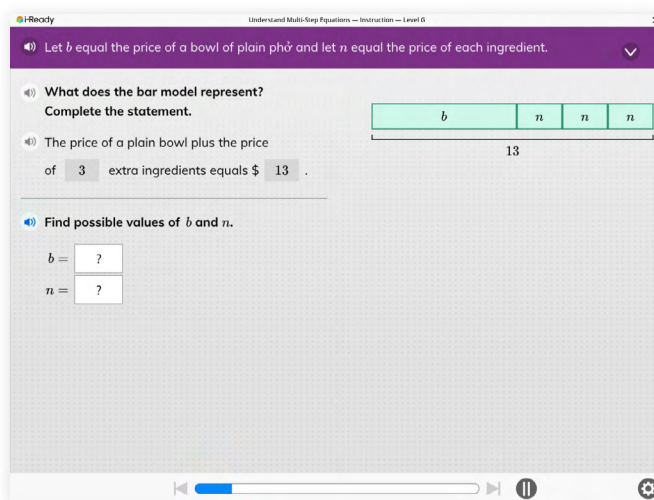
The National Research Council (NRC) (1989) found that traditional passive instruction often leads to mastery without deep understanding as students need support with retaining or applying their learning beyond standardized tests. By contrast, research shows that when students construct their own mathematical understanding by exploring, representing, solving, and proving, they develop a deeper grasp of concepts. Sfard (2003) emphasizes that instructional activities should be carefully sequenced, allowing students to first engage with concrete experiences and then explore the relationships between mathematical concepts. Hiebert (2003) further found that students in meaning-based programs not only understand underlying concepts but are also able to modify and invent procedures to solve new problems. Research-backed recommendations emphasize embedding mathematics in problem solving and engaging students as active participants who reason and communicate about mathematical ideas.

Elevate Lessons Promote Meaningful Learning

Elevate Lessons promote meaningful learning by integrating conceptual, factual, and procedural knowledge in problem-solving contexts. The lessons are designed to help students construct an understanding of mathematical concepts and procedures while recognizing the relationships between them. Using the *i-Ready* adaptive assessment, students are placed in a personalized learning pathway that ensures lessons build on their existing knowledge. Problem-solving challenges encourage students to practice familiar procedures while also exploring new concepts and approaches.

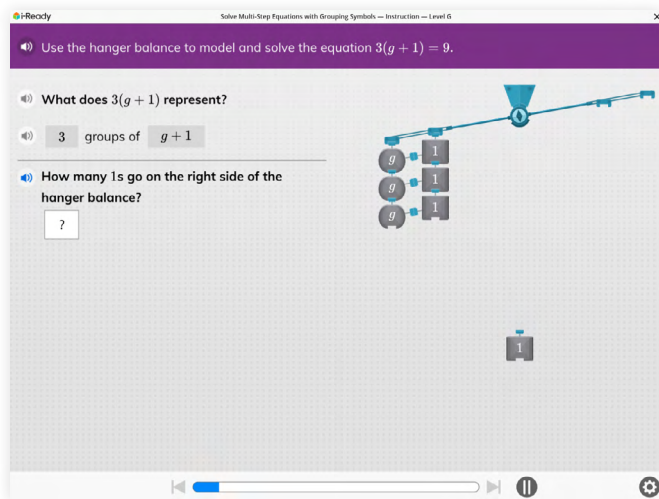
In Elevate Lessons, students take an active role in building meaning through open and guided explorations. These explorations allow them to make connections and develop deeper understanding of new concepts and procedures. Scaffolded instructional feedback is provided to support students' learning and ensure a deeper understanding of mathematical ideas.

For example, in the Understand Multi-Step Equations lesson, students first develop conceptual understanding through concrete explorations where they are tasked with making bowls of phở that meet certain criteria. They then connect this work to writing equations that represent similar situations.



Additionally, Elevate Lessons not only help students develop mathematical skills but also deepen their ability to recognize the structural relationships among concepts, properties, and procedures. By integrating conceptual understanding, factual knowledge, and procedural proficiency, the lessons ensure students see mathematical procedures as meaningful rather than as isolated steps to be memorized. This approach fosters a more connected and flexible understanding of mathematics, enabling students to apply their knowledge with confidence and adaptability in a variety of problem-solving situations.

For example, in the Solve Multi-Step Equations with Grouping Symbols lesson, students deepen their understanding of multi-step equations using hanger models. As the lesson progresses, they discover that these equations can be solved in different ways. They learn to identify appropriate approaches and have opportunities to decide how to begin solving the problem.



Solve Multi-Step Equations with Grouping Symbols — Instruction — Level 6

Use the hanger balance to model and solve the equation $3(g + 1) = 9$.

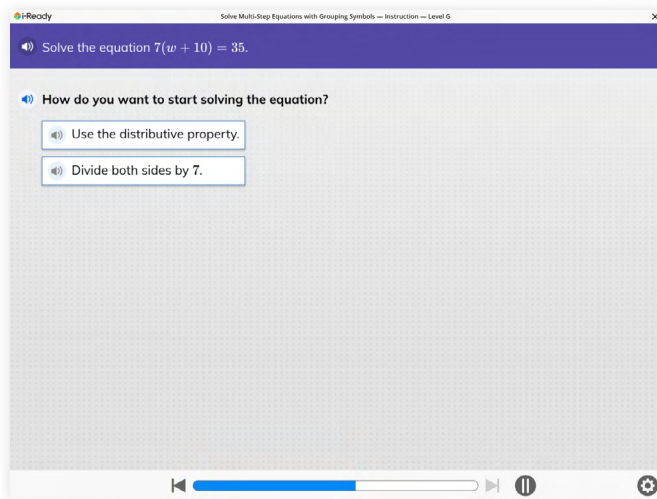
What does $3(g + 1)$ represent?

3 groups of $g + 1$

How many 1s go on the right side of the hanger balance?

?

The diagram shows a hanger balance with three groups of $g + 1$ on the left and one group of $g + 1$ on the right. The groups are represented by boxes labeled g and 1 .



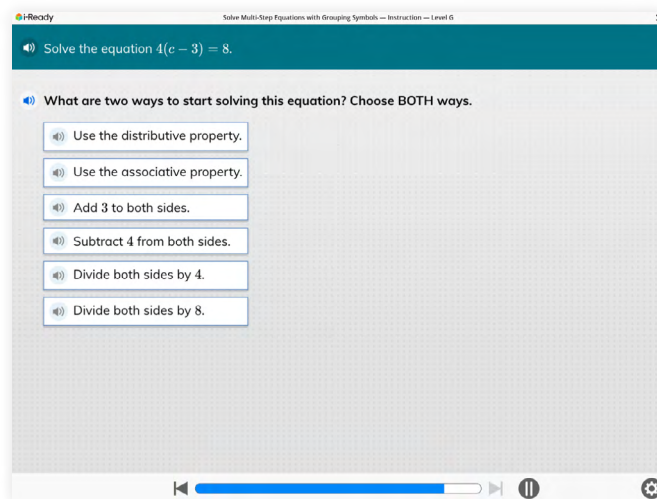
Solve Multi-Step Equations with Grouping Symbols — Instruction — Level 6

Solve the equation $7(w + 10) = 35$.

How do you want to start solving the equation?

Use the distributive property.

Divide both sides by 7.



Solve Multi-Step Equations with Grouping Symbols — Instruction — Level 6

Solve the equation $4(c - 3) = 8$.

What are two ways to start solving this equation? Choose BOTH ways.

Use the distributive property.

Use the associative property.

Add 3 to both sides.

Subtract 4 from both sides.

Divide both sides by 4.

Divide both sides by 8.

Effective Mathematics Instruction

Integration of Skills Development with Conceptual Development

Based on his review of empirical research on mathematics teaching and learning, Hiebert found that effective programs focus on both conceptual understanding and skills proficiency. He concluded that “both knowing and doing” are necessary for effective learning to occur. He advises that mathematics instruction should not abandon skills development but rather should incorporate it into meaning making through the construction of knowledge “while solving problems and [communicating their] ideas with others” (Heibert, 2003). Hiebert reported on studies comparing primary grade students who

participated in a year of instruction that promoted both conceptual understanding and skills development with students receiving instruction focused only on skills development. Students receiving the conceptual-plus-skills-development instruction demonstrated deeper conceptual understanding and were better able to develop new procedures or modify existing ones to solve novel problems. This greater understanding appeared to improve skills development rather than detract from it.

Noted mathematics educator Marilyn Burns agrees that understanding procedures, not just carrying them out, is essential for learning how to approach and solve problems in novel situations: “We must expect and demand that students learn to understand procedures, not only perform them. When their learning is based on understanding, students won’t be incapacitated if they forget a rule or step in a rule. Only with understanding will students be prepared to apply rules correctly in new situations” (Burns, 1998).

Diane Briars, former NCTM president, frames the need to integrate procedural knowledge with conceptual understanding in terms of workforce preparedness: “Being prepared for the 21st-Century workforce requires being able to do more than simply compute or carry out procedures. Children need conceptual understanding as well as procedural fluency, and they need to know how, why, and when to apply this knowledge to answer questions and solve problems. They need to be able to reason mathematically and communicate their reasoning effectively to others” (Briars, 2014).

Similarly, in his review of empirical research on mathematics teaching and learning, Hiebert (2003) found that traditional mathematics instruction focused on explaining, demonstrating, and discussing basic skills and procedures, along with student practice of those same skills and procedures. By contrast, alternative mathematics programs designed to promote more meaningful learning build “directly on students’ entry knowledge” and provide opportunities to solve problems that “require some creative work by students and some practice of already learned skills.” These programs emphasize comparing multiple methods for solving problems and ask students to explain and justify their solutions.

Impacts of Teaching for Meaning and Skills Development

In their examination of US mathematics education from Grade K to graduate school, the NRC, in conjunction with the Committee on the Mathematical Sciences, embarked on a multiyear project to identify strengths and weaknesses in the teaching of mathematics. In their report, they concluded that the traditional “passive” pedagogy ends up reinforcing mastery without understanding: “Students simply do not retain for long what they learn by imitation from lectures, worksheets, or routine homework. Presentation and repetition help students do well on standardized tests and lower-order skills, but they are generally ineffective as teaching strategies for long-term learning, for higher-order thinking, and for versatile problem solving” (NRC, 1989).

The NRC report points to research that shows that allowing students to construct their own understanding results in more effective learning: “Educational research offers compelling evidence that students learn mathematics well only when they construct their own mathematical understanding. To understand what they learn, they must enact for themselves verbs that permeate the mathematics curriculum: ‘examine,’ ‘represent,’ ‘transform,’ ‘solve,’ ‘apply,’ ‘prove,’ and ‘communicate’” (NRC, 1989).

Similarly, Hiebert’s (2003) review of mathematics teaching practices and learning revealed that with traditional pedagogy, students tend to learn the simplest knowledge and basic skills without much depth or conceptual understanding. Evidence for this was poor performance on items that require students to extend these skills, reason about them, or explain why they work. By contrast, Hiebert also found that with alternative programs that teach meaning, students constructed a deeper understanding of the concepts that underlie the procedures. This understanding showed itself in a variety of ways, including students’ ability to invent new procedures or modify old ones to solve new problems.

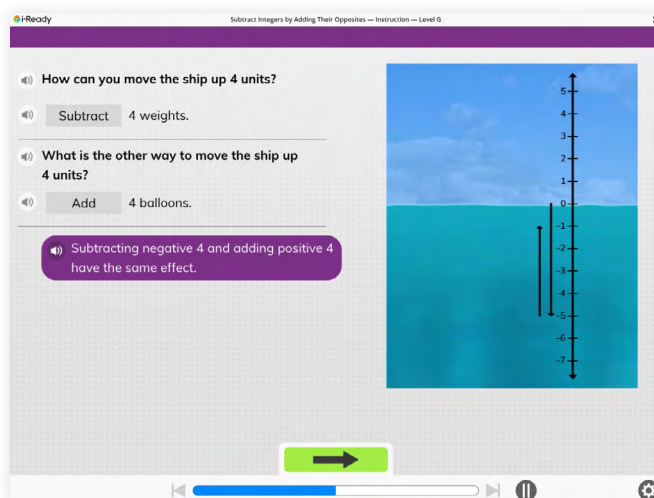
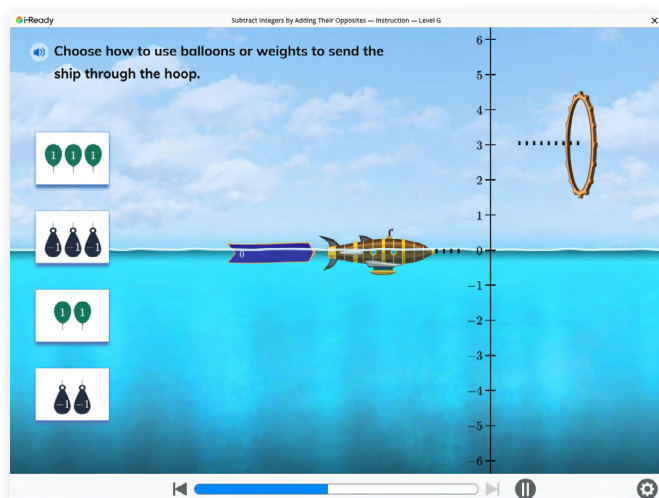
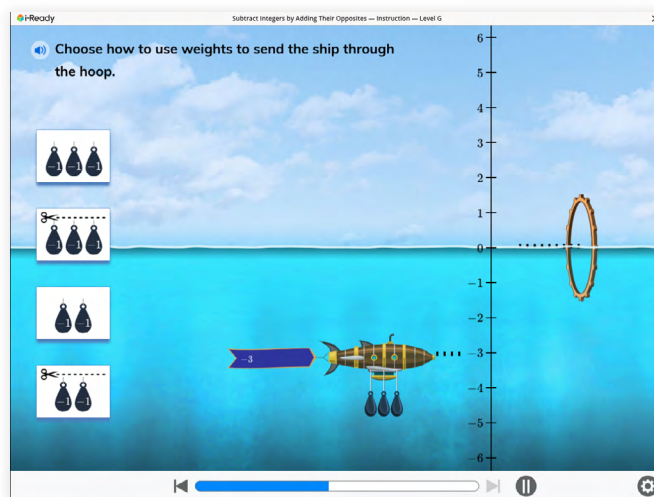
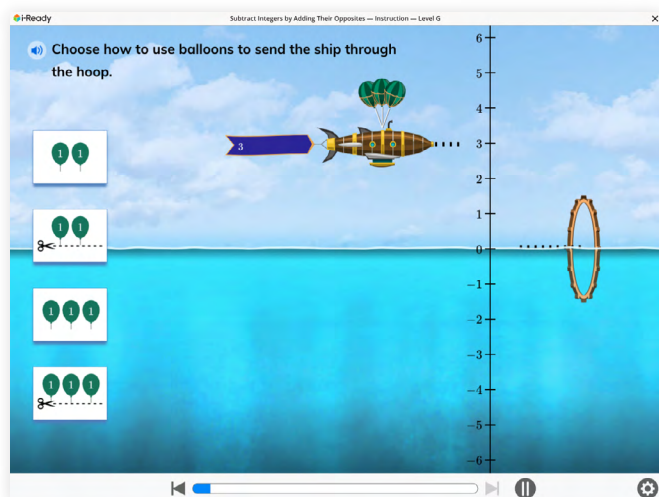
Attending to Recurring Procedures

Based on Sfard’s (2003) review of research and theory related to the NCTM Standards, along with critiques of the Standards, she concluded that in addition to focusing on meaning and structure, mathematics instruction needs to attend to “repetitive, well-defined actions”—in other words, recurring procedures. She argues that deep understanding of mathematics involves the study of repeatable actions. Learners must become sufficiently familiar with recurring procedures to make them more real—that is, “to be able to think and speak about the process in ways in which we think and speak about an object.” Sfard asserts that the NCTM Standards do not call for the abandonment of basic skills but rather “require that the skills be developed in new, more ‘natural’ ways.” However, she warns against a tendency for some educators “to interpret [the Standards] as denying the [basic] skills any real importance.”

Elevate Lessons Build Conceptual Understanding

Elevate Lessons allow students to develop effective strategies and procedural fluency while building a deeper conceptual understanding of the “why” behind the computations. The NCTM *Principles to Actions* framework emphasizes the need to build procedural fluency within the context of developing conceptual understanding, therefore building procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems (NCTM, 2014), therefore integrating conceptual understanding and skills proficiency.

For example, in the Subtract Integers by Adding Their Opposites lesson, students explore a context involving a ship navigating through rings. Through these explorations, they discover that adding balloons (i.e., positive integers) and subtracting weights (i.e., negative integers) have the same effect.



Research on Problem Solving, Modeling, and Representation

A substantial body of research shows that problem solving is a fundamental component of effective mathematics instruction at all grades. Engaging students in tasks that promote problem solving, reasoning, and deeper understanding creates a context in which fact memorization and procedural knowledge development can flourish and be meaningfully applied. Research also shows that for problem solving to be effective, tasks must provide an appropriate level of challenge and relevance for students. Additionally, effective mathematics instruction involves students in constructing and interpreting mathematical models—graphic and symbolic representations of problem situations—used as tools in problem solving.

Problem Solving in a Mathematical Context

Lesh and Zawojewski (2007) define mathematical problem solving as an iterative process of interpreting situations mathematically, involving cycles of expressing, testing, and refining mathematical ideas while integrating concepts from various areas within and beyond mathematics. Rather than viewing problem solving as a simple search for a procedure to move from a problem's "givens" to its "goals," they argue that problem solving should be seen as repeated cycles of deeper understanding of both the "givens" and "goals" (Lesh & Zawojewski, 2007).

Problem solving is one of five Process Standards in NCTM's Principles and Standards for School Mathematics (2000) and one of eight fundamental mathematical practices in NCTM's *Principles to Actions* (2014). The latter highlights the importance of tasks that "promote mathematical reasoning and problem solving," allowing for multiple entry points and "varied solution strategies" (NCTM, 2014). Burns (2015) further emphasizes that problem solving should be central to instruction, requiring students to go beyond finding correct answers by reasoning numerically, constructing arguments, communicating their ideas, and ensuring precision in their solutions.

Challenge and Significance

For problem solving in mathematics instruction to be effective, it must provide an appropriate amount of challenge. If problem-solving tasks are too easy or too difficult, learning is much less likely to occur (Sfard, 2003). Sfard connects NCTM's emphasis on aligning problem-solving lessons with a learner's maturity and prior experience to Vygotsky's work on keeping learners within their Zone of Proximal Development (Sfard, 2003). For Sfard, motivating a student's learning in mathematics requires consideration of both the difficulty of the problem and its significance within the learner's existing "system of concepts." She asserts that this connection between new learning and prior knowledge is supported by the work of Piaget and Vygotsky. Piaget defines learning as the process of "enriching and reorganizing existing mental schemes," whereas Vygotsky views knowledge development as emerging "through our constant dissatisfaction and incessant 'reworking' of what we already know" (Sfard, 2003).

Linking Problem Solving and Modeling

Effective mathematics teaching engages students in making connections among mathematical representations to deepen understanding of concepts and procedures and as tools for problem solving (NCTM, 2014). Mathematical modeling involves using structures such as graphs, diagrams, and equations to represent authentic real-world situations. These models or representations provide an abstract and simplified representation of the essential characteristics of a problem, making it easier to analyze and solve.

In their research review, Lesh and Zawojewski (2007) draw a strong link between problem solving and the development of mathematical models. They assert that students need to interpret problem situations mathematically in order to develop deep understanding of mathematical concepts, and such interpretation involves creating models. They also describe a process of learning mathematics through problem solving by creating, refining, and/or adapting interpretations embedded in mathematical models.

These researchers contrast this approach with traditional instruction, in which problem-solving strategies are typically introduced after procedures have been taught and authentic real-world problems are reserved for the final stages of instruction.

The Role of Representational Fluency in Learning Mathematics

Research indicates that developing representational fluency is essential for real-world applications. Representational fluency is the ability to transition smoothly among different representations of the same mathematical model (e.g., graphs, tables, equations, statements expressed in words) and to meaningfully connect these multiple representations. According to Lesh and Zawojewski (2007), representational fluency requires that students not only learn to use and understand representational media but also that they gain experience creating their own representations as part of problem-solving activities.

Research-Based Recommendations for Mathematics Instruction

Recommendations based on the research sources reviewed in this section about incorporating problem solving in mathematics education include the following:

- Problem solving should serve as a context for developing both conceptual understanding and procedural knowledge (Sfard, 2003).
- Instructional activities should engage students in solving problems that require both creative reasoning and application of previously learned skills (Hiebert, 2003; Sfard, 2003). The problems should encourage students to draw upon their existing system of concepts to make meaningful connections (Sfard, 2003).
- Problem-solving activities should strike a balance in terms of the level of challenge—offering enough challenge to promote conceptual understanding but not so much challenge to cause frustration. In this regard, individual differences among students should be taken into account (Sfard, 2003).
- Problem-solving activities should include representing problems with mathematical models, such as diagrams, graphs, and equations (Lesh & Zawojewski, 2007; NCTM, 2000; 2014). Over time, problem solving should help students develop representational fluency among different representational forms (Lesh & Zawojewski, 2007).
- Mathematics instruction should encourage analysis of multiple methods of solving problems and comparison of methods (Hiebert, 2003).
- Educators should encourage students to explain their problem-solving methods (Burns, 2015; Hiebert, 2003).

Problem Solving and Conceptual Understanding in Elevate Lessons

Problem solving is central to Elevate Lessons, promoting both conceptual understanding and real-world application. As students work through real-world problems, they first decontextualize by identifying or writing an equation or expression to represent the situation, or by using a visual model. After finding a solution, students recontextualize by interpreting their answer within the given context, sometimes adjusting numeric values to ensure they make sense in real-world scenarios (e.g., rounding 8.5 to 8 or 9 as appropriate).

For example, in the Write and Solve Addition Equations lesson, students begin by describing a real-world problem in words. They then complete an equation, substituting values for known quantities before solving.

The first screenshot shows the initial problem: "Nick has \$18.25 on his transit card now. He wants to add money to the card so that the final amount is \$30. How much money should he add?" Below the problem, it says "Describe the situation with an equation." There is a large empty box for writing the equation and a set of buttons for mathematical symbols (+, -, =) and labels (money on card now, money Nick adds, final amount on card).

The second screenshot shows the next step: "Let m be the amount of money Nick should add to the card. Complete the equation." The equation template is shown: $(\text{money on card now}) + (\text{money Nick adds}) = (\text{final amount on card})$. Below this, the variables are substituted: $? + m = ?$.

The third screenshot shows the final step: "Solve the equation." The equation is now fully substituted with the known values: $(\text{money on card now}) + (\text{money Nick adds}) = (\text{final amount on card})$
 $18.25 + m = 30$
 $m = ?$

Elevate Lessons also emphasize problem solving to deepen conceptual understanding. Lessons often begin with an exploratory activity or guiding questions that present a mathematical situation without direct explanation, allowing students to apply prior knowledge in new ways.

In Elevate Lessons, students are encouraged to explore multiple strategies for solving problems. They are introduced to various problem-solving methods and, when appropriate, given the flexibility to choose their own approach. This fosters deeper understanding and encourages them to experiment with different strategies to find effective solutions.

In Elevate Lessons, problem-solving activities are designed to provide both appropriate challenge and meaningful context. Lessons incorporate culturally and linguistically responsive content, ensuring that students see themselves reflected in the problems they engage with, for example by including diverse characters, varied interests, and highlighting culturally significant foods, activities, and celebrations. Additionally, Elevate Lessons encourage students to represent problems using mathematical models, providing tools that help them bridge the gap between concrete and abstract representations, deepening their understanding of mathematical concepts.

For example, in the Graph Systems of Linear Equations lesson, students are presented with a real-world problem about a student playing wheelchair basketball. They first write equations to represent the problem, then they decontextualize it by using defined variables. After solving, students recontextualize their solution by interpreting what the answer means in the context of the problem.


i-Ready Graph Systems of Linear Equations — Instruction — Level H

While playing wheelchair basketball, Orion scores 35 points with a mix of 2-point and 3-point baskets. He makes 15 baskets in all. How many 2-point and 3-point baskets does Orion make?

x : number of 2-point baskets
 y : number of 3-point baskets

Which equation shows the number of points Orion scores?
 $2x + 3y = 35$

Which equation shows the number of baskets Orion makes?
 $2x + 3y = 35$ $x + y = 35$
 $x + y = 15$ $2x + 3y = 15$



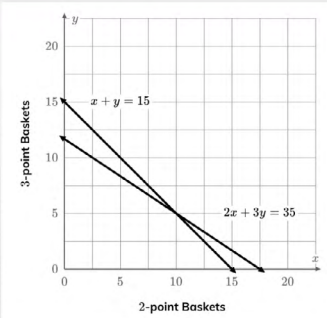
DONE

i-Ready Graph Systems of Linear Equations — Instruction — Level H

While playing wheelchair basketball, Orion scores 35 points with a mix of 2-point and 3-point baskets. He makes 15 baskets in all. How many 2-point and 3-point baskets does Orion make?

What is the solution to this system of equations?
 $(10, 5)$

How many 2-point and 3-point baskets does Orion make?
 Orion makes two-point baskets
 and three-point baskets.



CONCLUSION

Ensuring that all students, including those with unfinished learning from prior grades, receive instruction aligned with grade-level skills is essential for educational belonging. Research shows that students make significantly greater academic progress when given grade level-appropriate assignments (TNTP, 2018).

Elevate Lessons build on these findings by ensuring near- and on-grade level students receive instructional content that bridges learning gaps while keeping them engaged with grade-level coursework. Instead of traditional remediation, which can leave students perpetually behind, Elevate Lessons apply a tailored acceleration model. This approach uses data from *i-Ready* adaptive assessments, adaptive prerequisite reviews, and personalized just-in-time support to scaffold student learning without pulling them away from rigorous, grade-level instruction.

Rooted in the key mathematical domains outlined by NCTM and CCSS, Elevate Lessons deliver research-backed, engaging instruction in the Mathematics domains of Number and Operations, Algebra and Algebraic Thinking, Measurement and Data, and Geometry (NCTM & NCSM, 2020). The *i-Ready* adaptive assessment personalizes each student's learning path, ensuring lessons are built on prior knowledge while providing the appropriate level of challenge. Real-time data collection and feedback further refine instruction, ensuring students receive just-in-time prerequisite support without slowing their progress (NCTM & NCSM, 2020).

Designed to cultivate a deep understanding of mathematics, Elevate Lessons incorporate exploratory activities and scaffolded problem solving that encourage students to make connections to prior knowledge, fostering a growth mindset and the ability to succeed with grade-level content.

By modeling real-world situations, embedding culturally and linguistically responsive content, and integrating continuous assessment and support, Elevate Lessons ensure students not only develop critical-thinking and problem-solving skills but also gain the confidence and tools to succeed in rigorous mathematical coursework. Through innovative educational strategies and research-backed best practices, Elevate Lessons offer a comprehensive, equitable solution for learning acceleration—empowering students to close gaps, strengthen understanding of grade-level concepts, and achieve long-term success in mathematics.

REFERENCES

- Blackwell, L. S., Trzesniewski, K. H., & Dweck, C. S. (2007). Implicit theories of intelligence predict achievement across an adolescent transition: A longitudinal study and an intervention. *Child Development*, 78(1), 246–263.
<https://doi.org/10.1111/j.1467-8624.2007.00995.x>
- Briars, D. (2014). *Back to school: The time to engage parents and families*. National Council of Teachers of Mathematics. https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Diane-Briars/Back-to-School_-_The-Time-to-Engage-Parents-and-Families
- Burns, M. (1998). *Math: Facing an American phobia*. Mathematics Solutions.
- Burns, M. (2015). *About teaching mathematics* (4th edition). Mathematics Solutions.
- Center on Reinventing Public Education. (2022). *Back to school, but not caught up: Why accelerated learning only succeeds with strong systems*.
<https://crpe.org/report/back-to-school-but-not-caught-up>.
- Dweck, C. S. (2007). The secret to raising smart kids. *Scientific American Mind, Special Editions*, 23(5s), 76–83.
<https://www.scientificamerican.com/article/the-secret-to-raising-smart-kids>
- Hiebert, J. (2003). What research says about the NCTM Standards. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 5–23). NCTM.
- Lesh, R., & Zawojewski, J. (2007). Problem solving and modeling. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics*. NCTM.
- NCTM. (2000). *Principles and standards for school mathematics*. Author.
- NCTM. (2014). *Principles to actions: Ensuring mathematical success for all*. Author.
- National Council of Teachers of Mathematics & National Council of Supervisors of Mathematics (NCSM). (2020). *Moving forward: Mathematics learning in the era of COVID-19*. https://www.nctm.org/uploadedFiles/Research_and_Advocacy/NCTM_NCSM_Moving_Forward.pdf

- National Governors Association Center for Best Practices & Council of Chief State School Officers. (2023). *Common Core State Standards for mathematics*.
https://www.thecorestandards.org/wp-content/uploads/Math_Standards1.pdf
- NRC. (1989). *Everybody counts: A report to the nation on the future of mathematics education*. National Academies Press. <https://doi.org/10.17226/1199>
- New Classrooms Innovation Partners. (2019). *The iceberg problem: How assessment and accountability policies cause learning gaps in middle school math*.
<https://newclassrooms.org/wp-content/uploads/Iceberg-Problem-Full-Report.pdf>
- Robins, R. W., & Pals, J. L. (2002). Implicit self-theories in the academic domain: Implications for goal orientation, attributions, affect, and self-esteem change. *Self and Identity*, 1(4), 313–336.
- Rollins, S. P. (2014). *Learning in the fast lane: 8 ways to put all students on the road to academic success*. Association for Supervision and Curriculum Development.
- Sfard, A. (2003). Balancing the unbalanceable: The NCTM Standards in light of theories of learning mathematics. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 353–392). NCTM.
- Skemp, R. (1976). *The psychology of learning mathematics*. Routledge.
- Stipek, D., & Gralinski, H. (1996). Children's beliefs about intelligence and school performance. *Journal of Educational Psychology*, 88(3), 397–407.
- TNTP. (2018). *The opportunity myth: What students can show us about how school is letting them down—and how to fix it*. <https://tntp.org/publication/the-opportunity-myth>
- TNTP. (2021). *Accelerate, don't remediate: New evidence from elementary math classrooms*.
<https://files.eric.ed.gov/fulltext/ED615462.pdf>



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