Whitepaper

Supporting Productive Struggle for Mathematics Learners

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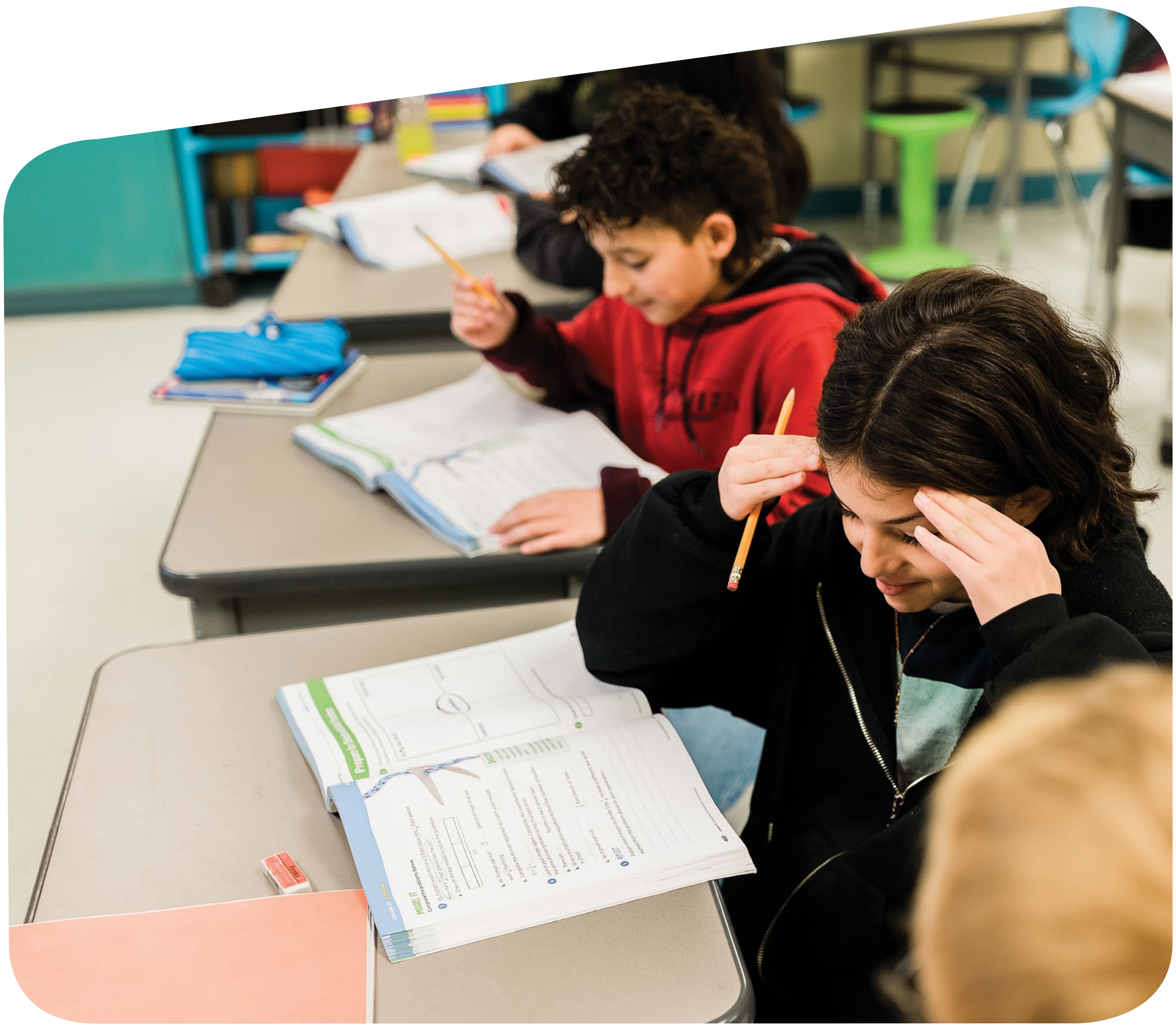

i-Ready Classroom Mathematics

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- Engaging and supporting all learners

Introduction

The Standards for Mathematical Practice (SMPs) (Common Core State Standards (CCSS), 2010) describe how students think and work as mathematicians. To develop these practices during instruction, we need to rethink how we engage students in learning mathematics. Fortunately, *Principles to Actions* (NCTM, 2014) includes eight Effective Mathematics Teaching Practices that help teachers engage students in the habits of mind highlighted in the SMPs (see Figure 1). An integral component of that work for students is to struggle productively. In this paper, we focus on how to support productive struggle in learning mathematics—a critical teaching practice.

All student and teacher practices work together to support learning. By effectively providing students with opportunities and supports to engage in productive struggle and sensemaking, both individually and collectively, teachers are laying the foundation for success for the other practices. Effective mathematics instruction includes engagement in productive struggle as students grapple with mathematical ideas and relationships and develop the habits of effective math students (NCTM, 2014).

Standards for Mathematical Practice (CCSS, 2010)

Mathematically proficient students:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

NCTM Teaching Practices (NCTM, 2014)

Effective mathematics educators:

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.

Figure 1. SMPs and NCTM's Effective Mathematics Teaching Practices

Struggle is at the heart of learning and thinking mathematically. As educators, we need to make sure we provide students opportunities to struggle in order to build their ability to productively engage in sustained mathematical thinking. Throughout this paper, we will highlight repeatable pedagogies and discuss manageable ways to promote productive struggle, including using routines, lesson frameworks, and basic teacher moves that can be implemented immediately and refined over time.

What Is Productive Struggle, and Why Is It Important?

Like us, you've likely seen when students have difficulties getting to a place of productive struggle. If they are overwhelmed or don't know where to start, they may shut down. Other times, they might engage productively until they get stuck and feel helpless unless they get explicit guidance from someone else. In both cases, they lose access to developing mathematical thinking.

Similarly, you've probably also seen unproductive struggle—when students work diligently but in a direction that is not fruitful. Struggle can also be unproductive when the content is out of reach for students and is non-existent if the task itself does not invite thinking and reasoning.

Productive struggle happens when students have a task or challenge that doesn't have an obvious solution path or procedure to follow (i.e., a high cognitive demand task). When students struggle productively, they:

- Make sense of an idea, situation, or problem
- Try to represent their mathematical ideas and strategies
- Consider and reconsider problem situations
- Attempt and reattempt problems
- Understand that errors and mistakes are part of the process
- Drive their own thinking through questions, multiple attempts, and collaboration with peers
- Explain why a strategy works or doesn't work in a situation

Students who regularly interact with high-demand tasks show greater learning gains than students who don't (Stein et al., 2009). That's why it's imperative to engage each and every learner in high cognitive demand tasks on a regular basis. In order for students to be able to transfer their mathematical thinking and learning to novel situations, it's critical that they engage in productive struggle throughout their learning process. When students don't engage in productive struggle, they may learn how to do the problem at hand but not develop their capacity to transfer that knowledge to unfamiliar situations. When teachers embrace productive struggle, it leads to long-term benefits, with students able to apply their learning to new problem situations (Kapur, 2010). In short, we need to ensure we are preparing students for the unknown world ahead.

In many other contexts, such as playing video games, sports, or learning an instrument, students already have growth mindsets. They understand that making mistakes is natural, and they learn from those mistakes . . . It's time to bring the same productive struggle that students apply to other situations to the mathematics classroom!

A critical part of productive struggle is believing that the process of working through ideas and mathematical challenges is helpful, doable, and will yield results. Students need to develop a growth mindset with the belief that because they can think and reason mathematically, they will ultimately make sense of an idea that might be challenging initially. Teachers need to help students see mistakes as valuable parts of the learning process, rather than as failures. In many other contexts, such as playing video games, sports, or learning an instrument, students already have growth mindsets. They understand that making mistakes is natural, and they learn from those mistakes. They realize that sustaining hard work yields improvement, and they see the fruitful results. It's time to bring the same productive struggle that students apply to other situations to the mathematics classroom!

A Culture for Productive Struggle

It is important to first establish a culture for productive struggle. For starters, ensure students are engaging with a task that will cause them to struggle—a cognitively demanding task is critical. Students will not need to struggle productively unless they have something to think and reason about. Start with a task that asks students to connect processes to underlying mathematical ideas or contexts, solve a non-routine problem, or make connections between representations.

Then, create a culture for productive struggle in which students feel safe and encouraged to share partially developed thinking, concepts as they are being built, and even incorrect answers. Such a culture emphasizes the value of co-constructing ideas and collaboration. Solving complex problems is most efficient when the process includes multiple perspectives and a range of thinking. As a result, teachers and students alike will internalize the idea that developing thinking takes time, multiple drafts, and even multiday experiences to develop a mathematical concept. A prominent overarching message in classrooms in which students struggle productively is that **mistakes are valuable**. Although we often tell students that mistakes are valuable and include that message on a bulletin board or poster, it's not enough—we must build this belief through classroom culture as well.

While students commonly accept that other subject areas require multiple drafts for essays and papers, they don't often draw the parallel to mathematics. Mathematical thinking also needs to be revised or refined (Jansen, 2020). When students understand the need for revision, they are more open to productive struggle and learn to analyze their own mistakes, each other's mistakes, and even errors that appear in worked examples or error analyses. In addition to supporting productive struggle, the work of naming an error or misconception and justifying it mathematically allows students to make connections and develop long-lasting learning.

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Four Key Elements to Engage Students in Productive Struggle

1. Provide time for students to develop their thinking and questions.

Students need time to make sense of a problem situation, to formulate their own thinking and questions, and to process each other's ideas. After providing individual think time, students then benefit from time working through ideas with a partner. While we know time is critical to developing productive struggle, and that productive struggle is critical to developing conceptual understanding and mathematical reasoning, the urgency of curriculum demands sometimes causes teachers to rush through this phase of learning. Slowing down and providing time for students to develop their thinking is important so they can internalize their learning.

2. Provide multiple ways for students to process and reprocess ideas.

Students need multiple ways to process and reprocess information and ideas in order to struggle productively and refine their understanding. They will likely need time to think on their own, talk through ideas with a partner, or reference visuals to support their thinking and prepare them to communicate ideas. While students work, consider providing them with a variety of tools to choose from, including colored pencils, manipulatives, technology, etc. During classroom discussions, have students restate or rephrase what others say to allow students to process and reprocess new ideas or the ideas of others. Prompt turn and talks to provide a structure for students to work out ideas in a safe space.

3. Ask purposeful questions that focus student attention.

Throughout the lesson, students will likely need support to jumpstart or push their thinking. However, supporting students with productive struggle is vastly different from supporting students with getting to an answer. Teacher support entails timely and purposeful questions that focus student attention or reorient them to the goal of the lesson. It doesn't mean a series of pointed questions that funnel student thinking to a desired result. Instead, teachers listen to how students are thinking and then pose a question that acknowledges that thinking and reorients it.

After asking a question, the teacher then steps away without waiting for a response to allow the students to continue doing the intellectual work of the lesson.

For example, a teacher may notice students creating long lists of trial and error to solve a problem and say something like, "I see you've tested a whole bunch of numbers. Now, take a step back and see if you are working with the numbers in the same way each time. That might help you generalize this process." Students may also think they've found a valid strategy, so the teacher asks, "Will that always work? How do you know?" In either case, after asking a question, the teacher then steps away without waiting for a response to allow the students to continue doing the intellectual work of the lesson.

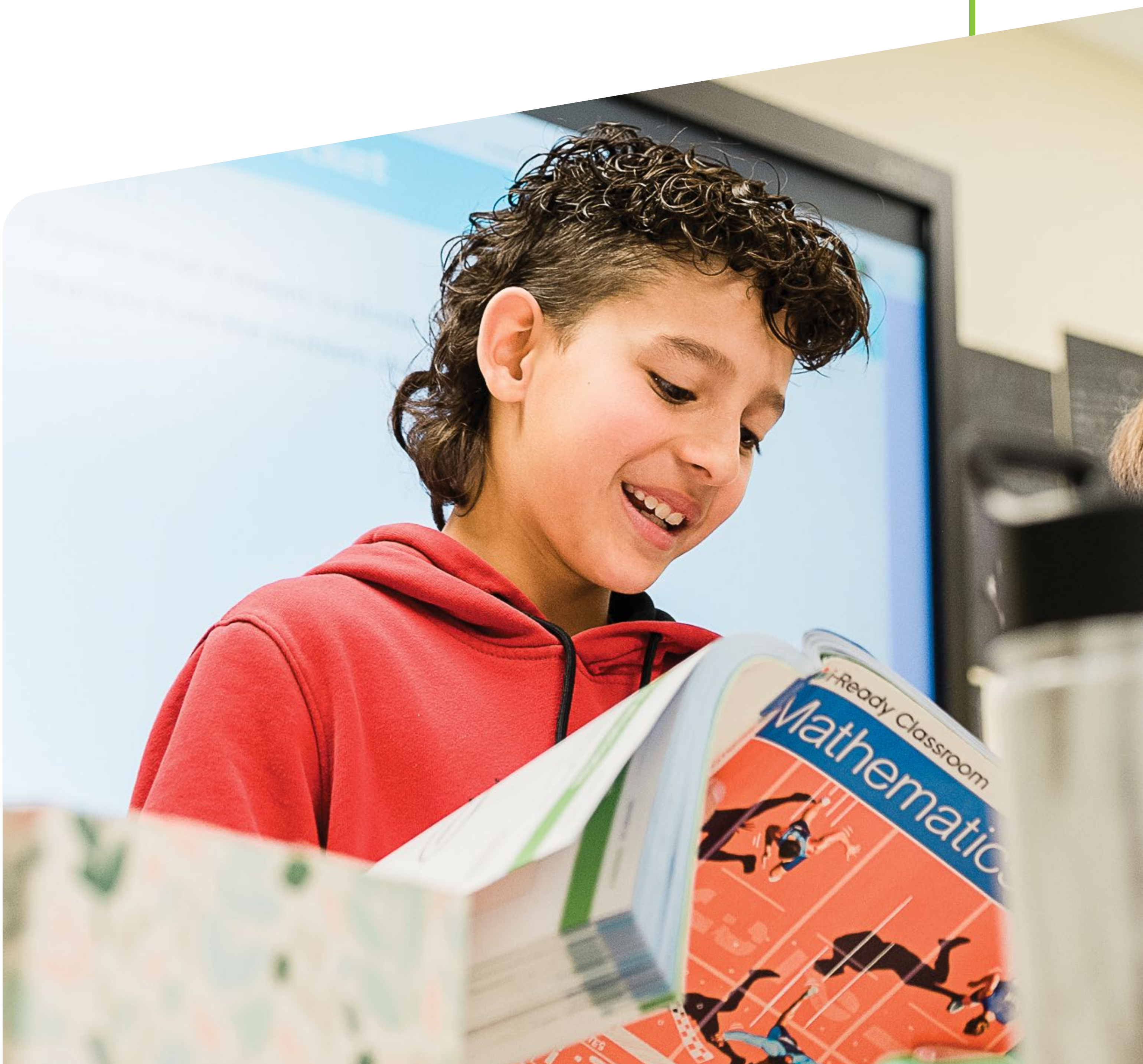
Over time, the questions teachers ask regularly will become prompts for students to ask themselves (i.e., Ask Yourself Questions) to support students in productive struggle even when the teacher is not there (Kelemanik & Lucenta, 2022).

4. Explicitly focus on sensemaking processes.

Throughout the lesson, continue to emphasize strategies to struggle productively by highlighting sensemaking processes. For instance, after the class has interpreted a problem situation together by reading it multiple times and identifying important information and relationships, ask students to reflect on that process. Name what was helpful about the sensemaking process so they can apply it again in other situations.

Be explicit when engaging students in sensemaking strategies like noticing and wondering so they learn to apply that strategy on their own when approaching complex mathematical situations.

Similarly, make Ask Yourself Questions concrete for students so they recognize it as a repeatable support to jumpstart their thinking.



Critical Points in a High-Demand Lesson to Support Productive Struggle

To support students with productive struggle, it's important to know when they are likely to struggle. Consider a typical flow for a problem solving–based lesson. Students often make sense of a problem situation, work independently, or work with a partner to solve the problem, then discuss and connect classmates' approaches and solutions. Students may get "stuck" and fall out of productive struggle in each of these phases of the lesson.

How do you integrate supports during these critical times in the problem-solving process to encourage continued productive struggle? We'll look at strategies to help students persevere during each possible obstacle of the problem-solving process.

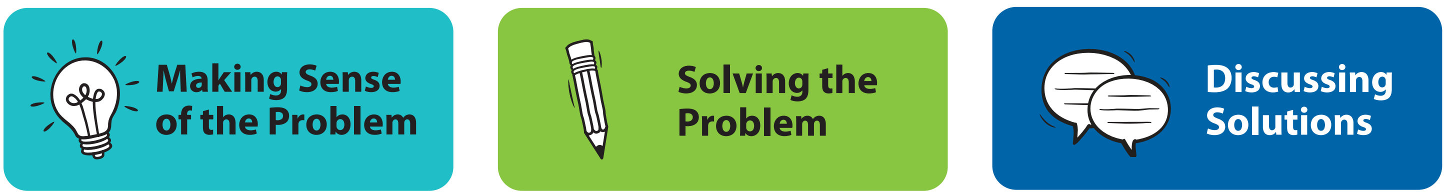


Figure 2. Critical points in a high-demand lesson where productive struggle may end

SUPPORTING PRODUCTIVE STRUGGLE:

Making Sense of the Problem

Students often jump in quickly to solve a problem without carefully reading it. Teachers who support productive struggle encourage students to slow down. They take time to teach students how to make sense of a problem before they try to solve it. Clarifying any contextual barriers is important so students can determine what mathematics might help them solve the problem. Prompt students to consider the context, the question being asked, and the important information so they're clear on what is being asked, what information is given, and what assumptions are being made before selecting a solution strategy. In early grades (i.e., Grades K–2), teachers may share a story situation, act out the context, and/or have students draw a picture to support their sensemaking process. These strategies help students develop habits like pausing and interpreting problem situations as the first part of solving a problem, thereby setting them up for success in future problem solving. In this way, teachers who support productive struggle can help students process the information in a problem without telling them how to solve it or offering a strategy. Let's look at an example from Grade 4 in *i-Ready Classroom Mathematics* (see Figure 3).

Read and try to solve the problem below.

Jaime finds 3 times as many shells at the beach as Calvin finds. Jaime finds 24 shells. Write and solve an equation to find the number of shells Calvin finds.

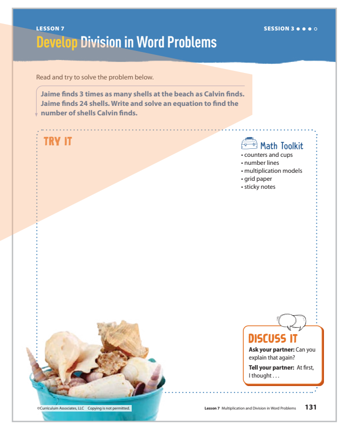


Figure 3. Problem from *i-Ready Classroom Mathematics* Grade 4, Lesson 7

Analyzing Quantities and Relationships

All too often students dive into a problem like the one shown above by grabbing numbers and operating without considering the important mathematical relationships. Regarding the problem in Figure 3, students are likely to read this short prompt, focus on the two numbers—3 and 24—and the keyword—times—and jump right to multiplying 3 x 24. Instead, they need to pause to make sense of the problem first. One way to do this is to **have students identify important quantities and relationships in the problem**. When students slow down, they can identify important quantities and relationships. Modeling this together as a class helps students learn to do this on their own.

What Quantities Are in the Problem?	What Relationships Are in the Problem?
The number of shells Jaime found (Jaime has 24 shells.) The number of shells Calvin found (We don't know this number yet.)	The number of shells Jaime found is three times the number of shells Calvin found. (Jaime has three times as many shells as Calvin.)

Figure 4. Identifying important quantities and relationships in problems

Without making sense of the problem, students can not only get an incorrect answer but also entirely miss the mathematical point of the lesson: *to multiply or divide to solve word problems involving multiplicative comparisons (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison* (CCSS, 2010)). Position students to struggle productively by pausing and orienting them to the important information by prompting them to name the quantities in the problem statement, describe in their own words how those quantities are related, and ask (if needed), "How many shells does each student have?" before allowing them to represent and solve the problem.

Making Sense of Problems Using the Three Reads Routine

Sometimes students read a word problem, can't immediately make sense of it, and shut down. Or, worse, they don't even read the problem because they see what they think are "too many words," assuming from the start that they will be unable to understand it, let alone solve it. You can help students shift their unproductive belief that they "can't do it" and give them strategies that will allow them to think through a problem. This is especially important with problems that are lengthy, contain a plethora of information, have a context that might be unfamiliar to some students, or don't have a suggested or well-rehearsed solution method. Problems like this take time and require multiple reads to process (Kelemanik et al., 2016). A teacher who is supporting students to productively struggle in solving problems starts by helping them make sense of a problem. One way to do this is by using the Three Reads routine. The problem in Figure 5 illustrates the Three Reads routine to make sense of a complex problem.

> Noah designs a set for a school theater production. He has 150 cardboard bricks. He uses some of the bricks to make a chimney. To make an arch, he needs to use 4 times as many bricks as he uses for the chimney. He also saves 15 bricks in case some get crushed. How many cardboard bricks can he use to make the arch?


LESSON 18 | SESSION 2

Develop Writing and Solving Equations With Two or More Addends

Figure 5. Problem from *i-Ready Classroom Mathematics* Grade 7, Lesson 18

Using Three Reads to Make Sense of a Problem
(Modeled Using the Problem in Figure 5)

	After reading the problem . . . (Sample responses for the problem in Figure 5)
Read 1 **Teacher reads the problem.** Focus only on understanding the context and key vocabulary of the problem.	• Ask a few students what the problem is about. (A theater production, a school play, a set for the play, the scenery for the play, an arch, a chimney, bricks) • Clarify any unknown words and critical vocabulary. (What is a set for a play? How would you describe an arch?) • Have students talk about any visuals with the problem. (Noah is putting up bricks in an arch. There are four times as many bricks for the arch, which is a rounded doorway.) • Discuss prior experiences with the context. (Have any students been to a theater or seen a play? Have any students created a set for a play?)
Read 2 **A student volunteer reads the problem.** Focus on helping students understand what they are trying to find out (but not how they will find it).	• Ask students to restate the question in their own words. While the question may be straightforward, reading the problem a second time and prompting students to rephrase the question will help them clarify the information and orient to an implied quantity. (How many bricks does he have to make the arch after the chimney is made and 15 bricks are put aside?)

Read 3
The class reads the problem together.
Focus on what information is important and identify known and unknown quantities and relationships.
	• What quantities (i.e., numbers representing a count of an object) are in the problem?
	  • The number of bricks used for the arch (?)
	  • The number of bricks used for the chimney (?)
	  • The number of bricks saved in case some break (15)
	  • The total number of bricks (150)
	• What relationships (i.e., comparisons between quantities) are in the problem?
	  • The number of bricks used for the arch is four times the number of bricks used for the chimney.

Notice that the teacher does not explain and tell but rather provides space and prompts for the students to make sense of the problem. It's important that during the "make sense" portion of the discussion, students and teachers don't discuss how to approach a solution to the problem (i.e., "We're going to divide") as students may approach the problem in many different ways.

The goal of the Three Reads—or any strategy used to support students as they work to make sense of a complex math problem—is to build the mathematical habit of reading and rereading to ensure they understand the context, know the question(s) being asked, and have begun to identify important information, both known and unknown. This not only helps students make sense of one problem, but also helps build their capacity to make sense of future problems. When students have a strategy for making sense of problems, they are more likely to try problems, productively struggle, and not give up.

Teacher Tips: Making Sense of the Problem

1 Identify Possible Obstacles in Advance

Read and reflect on the problem in advance. Ask yourself, "Is there vocabulary or something about the context my students will not understand that's necessary in order to solve the problem?" If the answer is yes, be prepared to support students by asking the class to clarify certain language (e.g.,"What's a set in this situation?") or about their experience with the context. For less familiar contexts, you may want to plan to show a video, use pictures, or provide other supports, and decide in the moment whether or not students need them. Often students surprise us with the background and knowledge they bring.

2 Take the Time to Make Sense of the Problem

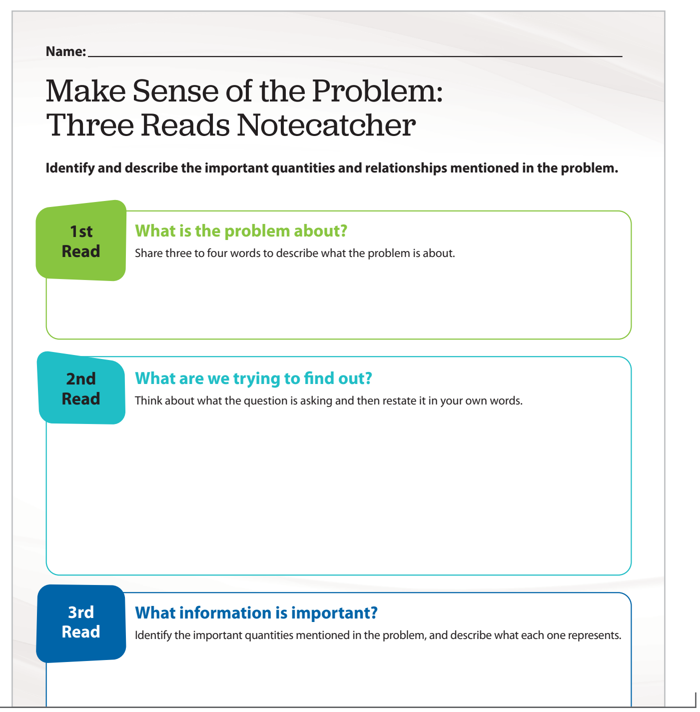
Build in time for students to wrap their head around the task before moving to solving the problem. Ask yourself questions such as, "Are my students likely to number grab?," "Would students benefit from talking through the problem with a partner?," and "Should we discuss and record information together as a class?" If the answer to any of these is yes, decide what structures you will use to ensure your students have the time they need to make sense of the problem.

3 Say Less When You Launch the Task

Set students up to be the ones doing the thinking. Ensure the prompts you give them spark their thinking, and don't inadvertently tell them what or how to think by offering a method or strategy. Ask yourself, "Will the question I'm going to ask or the prompt I am going to give position students to think and reason, or will it tell them what to do?" If the question or prompt guides students on what to do, then rephrase.

4 Support Students in Making Sense When Working Independently

It's important to encourage the process of making sense of problems when students are working independently. Students are more likely to persevere when they have a known starting point. After engaging them in the process, you may want to give students a way to capture their sensemaking with a document like the Three Reads Notecatcher shown in Figure 6. You may want to collect students' notecatchers to show that you value how they make sense of the problem, not just the solution to the problem. After doing this together as a class for a little while, you can slide the notecatcher into a plastic sleeve so students can use it with a dry-erase marker for problems in class or at home.


Name: ______

Make Sense of the Problem: Three Reads Notecatcher

Identify and describe the important quantities and relationships mentioned in the problem.

1st Read — **What is the problem about?** Share three to four words to describe what the problem is about.

2nd Read — **What are we trying to find out?** Think about what the question is asking and then restate it in your own words.

3rd Read — **What information is important?** Identify the important quantities mentioned in the problem, and describe what each one represents.

Figure 6. A sample Three Reads Notecatcher available for *i-Ready Classroom Mathematics* users on *i-Ready Success Central*

SUPPORTING PRODUCTIVE STRUGGLE:

Solving the Problem

Teachers who support productive struggle allow students time to think on their own about how they might approach a problem or task. But sometimes students can get stuck, start spinning their wheels, or go down a non-productive path. To make sure students' struggles are productive, or to help students **get unstuck** or **redirect their thinking**, provide multiple processing structures and ask questions that orient student attention. Most critically, do this without taking over student thinking.

When students get stuck, how can you help? The key lies in asking a question that will get them thinking—and then walk away. Let's explore this idea more by looking at the sample Grade 3 problem shown in Figure 7.

Read and try to solve the problem below.

Malik has 205 seeds. He plants 137 seeds with his stepdad. How many seeds does Malik have left?

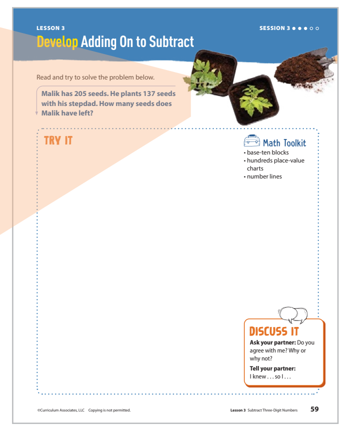


Figure 7. Problem from *i-Ready Classroom Mathematics* Grade 3, Lesson 3

The goal of this lesson is to "fluently add and subtract within 1,000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction" (CCSS, 2010). Imagine you hear the following from a student working on the problem who is stuck: "I know I have to subtract 137 from 205 to get the answer, but I don't know how." You could stop to explain how to subtract with regrouping or suggest another subtraction strategy, but that would be you doing the thinking, and the student would need to process your thinking. Instead, you might say, "Think about a way you could represent the problem. What tools or representations have you used before when subtracting?" Then walk away, giving the student time to think.

What if the student isn't stuck but is using a less-than-productive approach that has distracted them from the mathematical goal of the lesson? For example, you observe a student drawing 205 seeds with the intention of crossing off 137 of them. You might acknowledge that their strategy will work—if they are careful—and then ask a few questions, such as, "What operation are you representing?" or "Is there another tool or representation you could use to subtract 137 from 205?" Again, walk away after posing the prompt.

In both cases, the teacher move was to spur the student back into productive struggle by prompting an alternative way to represent or think about the mathematics without telling them which representation or approach to use. Walking away after asking students a guiding question keeps the teacher from getting pulled into providing too much help and more importantly sends the message to the student that they are capable of solving the problem on their own.

Individual think time precedes partner work and sets students up to solve a problem productively together. Giving students individual think time to consider how they might approach a problem not only allows them to identify areas of confusion and/or questions they have, but it also prepares them to work productively with a partner. By giving students time to talk with a partner, they continue to productively struggle instead of getting stuck and disengaging.

Partners provide other perspectives and approaches to consider as well as the opportunity to ask questions and explain and defend one's own thinking. All of this helps students gain clarity and deepen understanding while staying productively engaged with the task at hand. It also makes student thinking audible so the teacher can get a better understanding of how students are making sense of the mathematics. The teacher can hear if and where they are getting stuck, moving away from the lesson's mathematical goal(s), or developing ideas that should be discussed with the whole class.

Launching partner conversations in a way that will encourage students to contribute and share ideas is important. Discussion prompts help focus students' conversations on the mathematics and give each partner a place to begin the conversation, especially if they are uncertain how to start. This is especially helpful for students who are hesitant to share their ideas or students who are multilingual learners.

Some sample sentence frames to use as partner discussion prompts are shown below in Figure 8.

Figure 8. Sample partner discussion prompts taken from
i-Ready Classroom Mathematics Discourse Cards

Teacher Tips: Solving the Problem

1 Ask Students to Think about a Visual or Tool That Will Support Their Understanding of the Problem

A picture or manipulative tool can offer students another way to "see" a possible strategy for solving the problem. This can be especially helpful to students who benefit from multiple representations or visuals, including multilingual learners and students with learning disabilities.

2 Balance Individual and Partner Time

It's important to transition to partner conversations before students have completely solved the problem. This sets students up to talk about their thinking and approach, not just share their answer.

3 Provide a Sentence Frame or Sentence Starter to Begin Partner Discussions

Sentence frames or starters are crucial to focusing the conversation. They help students collect their thoughts and help those who are less comfortable talking begin a partner conversation.

4 Use a Variety of Ways to Partner Students

Use partnering routines (e.g., clock partners, shoulder buddy, etc.) to strategically support student pairs. For example, if you want a particular student to begin sharing with their partner first, give a direction that targets that student (e.g., "The person in the pair closest to the windows will begin by asking . . ." or "The partner whose head is closer to the ceiling will share their thinking first using the sentence frame . . .").

5 Ask Students Questions to Help Redirect Their Thinking without Telling Them What to Do or Think

It's important to ask students questions that support them and get them thinking without telling them what to do. To encourage student thinking, step away after asking a question and let students think about what you asked them. Use this strategy when students are working on their own or with a partner. Sample questions include:

- "What do you know about the quantities we've named?" while pointing to the list generated
- "What relationship(s) can you describe between any quantities?"
- "What do you notice [in the visual/problem, etc.] that might be mathematically important?"
- "What's another way you could write/express this [number, operation, expression, equation]?"
- "If you drew a picture to represent the situation, what would that look like?"
- "What mathematical tools could help you think about this problem?"

SUPPORTING PRODUCTIVE STRUGGLE: Discussing Solutions

As multiple students share their thinking with the class, it's important to engage the rest of the class in making sense of what's shared. This way, teachers have the opportunity to support perseverance in all students as they work together to understand others' ideas and make connections among strategies.

These multiple opportunities for students to interpret and make sense of student-generated work positions everyone in the class to dig into deeper ideas and allows the teacher to pose questions to further their understanding. This approach encourages students to productively struggle to understand not only their way of thinking but also the thinking of others. This is in sharp contrast to having students stand up and present their work—often focusing on the answer they got rather than explaining and justifying their thinking—and then having the teacher thank them for sharing.

Let's dive deeper into how to support productive struggle during whole class discussions using some specific problems. We'll start with an example of a Grade 1 task that prompts students to consider another's thinking (see Figure 9).

? There are 38 baby puffins.
There are 51 adult puffins.
Are there more adult or baby puffins? Show how you know.

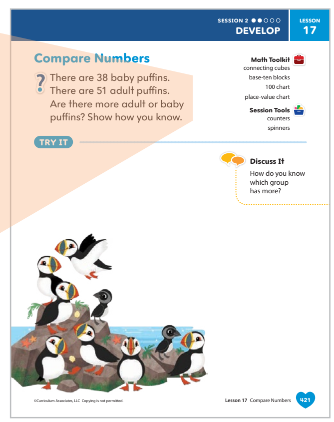


Figure 9. Problem from *i-Ready Classroom Mathematics* Grade 1, Lesson 17

As students work on this problem, they are comparing the number of baby puffins and the number of adult puffins. After using the Three Reads and giving students some time to think about the problem alone, Ms. Garcia prompts students to share their thinking with a partner. She listens to partnerships around the room and pulls the class back together to discuss. The conversation may sound like the dialogue on the following page.

Discussing Strategies as a Class

"So, some of our friends think there are more adult puffins, and other friends think there are more baby puffins, and some are still deciding. When we don't all agree, it's always helpful to discuss our ideas. I've asked Peter to share his reasoning to start us off."

Ms. Garcia

"I wrote down 38 and 51 to compare them, and I see 8 is more than 1, so that's why I think there are more baby puffins."

Peter

"Who can say what Peter just said, but say it in their own words?"

Ms. Garcia

"Peter first wrote the two numbers down, and he thinks there are more babies because 8 is greater than 1."

Maria

[Ms. Garcia records the two numbers 38 and 51 and the number sentence 8 >1 as Maria says it. She then pauses for about five seconds for students to see what she wrote.]

"In just a second, I'd like you to talk to your partner about this number sentence. Will it help you decide if there are more adult or baby puffins, and how do you know? Take one minute to discuss with your partner."

Ms. Garcia

[Ms. Garcia listens to student pairs and decides to call on Alicia and Kelly to share their thinking.]

"Kelly and I think there are two numbers, not just an 8 and a 1. So, we should use 51 and 38 to see that there are actually more adult puffins because 51 is more than 38."

Alicia

"Who can add on to Alicia and Kelly's thinking?"

Ms. Garcia

"We have to look at the entire number, not just the ones place. 50 is way more than 30, so there are definitely more adult puffins."

Ryan

"I also respectfully disagree with Peter because he is looking at the ones, and you have to look at the tens."

Samantha

Discussing Strategies as a Class, Cont'd.

"Who can restate what Samantha just said?"

Ms. Garcia

"Samantha said that we have to start by looking at the tens, not the ones. If we only look at the ones, we won't know which number is bigger."

Ana

"Turn and talk to your partner. How might looking at the tens help decide whether there are more adults or baby puffins?"

Ms. Garcia

[After giving a little time for students to think and talk, Ms. Garcia selects a student to share their thinking.]

"There are 38 baby puffins, and the first number tells us how many tens. So, the babies have 3 tens and the adults have 5 tens, which is more."

Soham

"Let's check back in. Has anyone changed their thinking? Show me a thumbs up if you think there are more baby puffins, thumbs sideways if you're not sure, and thumbs down if you think there are more adult puffins."

Ms. Garcia

[Ms. Garcia scans the room and sees that all students have their thumbs down.]

"If you changed how you were thinking, what was most helpful?"

Ms. Garcia

"It helped to write down the number sentence."

Samantha

"It helped to hear from classmates."

Peter

"When Alicia and I were talking, we thought about base-ten blocks."

Kelly

"Great thinking, everyone!"

Ms. Garcia

Let's return to Noah's school theater problem (see Figure 10) and use it to discuss additional ways to support students with productive struggle when discussing solutions.

> Noah designs a set for a school theater production. He has 150 cardboard bricks. He uses some of the bricks to make a chimney. To make an arch, he needs to use 4 times as many bricks as he uses for the chimney. He also saves 15 bricks in case some get crushed. How many cardboard bricks can he use to make the arch?

Figure 10. Problem from *i-Ready Classroom Mathematics* Grade 7, Lesson 18

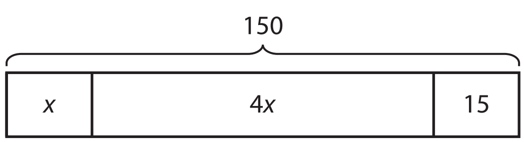
Once students make sense of the problem (as described on pages 11–13), they work on this task individually, then discuss their strategies with a partner. After this discussion, they are ready to interpret and discuss student-generated solutions and/or strategies provided in the text. It's vital that students have time to process and work through the task individually and/or with a partner before interpreting someone else's thinking.

Students may be asked to analyze a strategy (either from another student or the text) like the one shown in Figure 11. If teachers start asking questions too soon—before students have had time to make sense of the strategy—students will not be prepared to respond or contribute. As a result, they may feel that their only option is to co-opt another student's thinking. To support productive struggle, it's important to provide students with private think time to make sense of the strategy before asking questions.

Model It

You can draw a bar model to make sense of the problem.

Let x represent the number of bricks in the chimney.



Use the model to write an equation.

$$x + 4x + 15 = 150$$

$$5x + 15 = 150$$

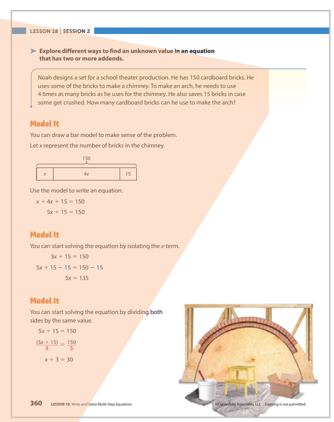


Figure 11. A sample solution strategy for Noah's problem from *i-Ready Classroom Mathematics* Grade 7, Lesson 18

A Structured Process for Analyzing Strategies

1. Individual Think Time

Give students time to make sense of the visual, the surrounding annotation, and equations. You may want to break this into sections if students have trouble processing too much at one time. Prompts may sound like:

- "Take a minute on your own to look at the model. What do each of the numbers and variable expressions represent?"
- "Take 30 seconds of private think time. Ask yourself, 'How are the numbers from the problem shown in the model?'"
- "Individually, consider the model. How are the quantities and relationships represented in it?" (Described on page 11.)

2. Turn and Talk

Prompt students to think further about the visual by posing a question and asking them to turn and talk about it. Ask a question like:

- "How does the bar model represent the situation?"
- "Why is it useful to use x to represent the number of bricks in the chimney when the problem asks for the number of bricks in the arch?"

3. Connect Representations

Students then work together to contextualize aspects of the bar model and connect it to the quantities in the problem situation, thereby deepening understanding of the equation that follows and the solution to the problem. Ask a question like:

- "How does the model connect to the first equation?"
- "How does the equation represent the situation?"
- "When you solve the equation, what will the value of x tell you?"
- "Are you done solving the problem when you find the value of x? Why or why not?"

After students analyze classmates' strategies and/or text-based approaches, teachers can continue to promote productive struggle by highlighting ways to make sense of problems that can be applied to future problem situations. Explicitly reflecting on and naming mathematical thinking will support students as they encounter novel situations in the future. Repeated use of the Three Reads Notecatcher (see Figure 6 on page 14) and analyzing quantities and relationships in problems (see Figure 4 on page 11) will help students persevere.

At the end of the discussion, students may be asked to look back and consider the models or strategies they and the class developed and discussed. Ask them to name which strategies they find most effective and to explain their reasoning for selecting them. Taking the time to do this analysis will help students next time they are faced with a similar type of problem.

Teacher Tips: Discussing Solutions

1 Give Students Opportunities to Analyze Work That Is Both Correct and Incorrect

It is important that students analyze others' work often and not just when work is correct. Sometimes the work may come from students in the classroom, and sometimes it is crafted specifically to highlight a specific strategy or a common misconception. When students analyze others' work, it:

- Sends the message that they can learn from each other and from mistakes, and it sets a norm for doing so
- Prepares students to respectfully look at classmates' work and learn from it, whether it's completely accurate or not, and it builds a safe environment to do so
- Helps students develop a critical eye when interpreting others' strategies and an appreciation for the learning opportunity that analyzing errors provides

2 Give Students Time to Process Others' Ideas Individually and with Partners

Sharing strategies shifts student attention from answers and procedures to explanations and justifications. Initially, when student-generated work is shared, classmates need time to process the ideas, representations, and calculations. Teachers support productive struggle by providing private think time and an opportunity for students to talk to a partner to articulate their interpretation of other students' work or pose a question about the work before discussing the work in the whole class.

3 Focus on the Thinking, Not Just the Answers

Prompt students to explain their own thinking, justify their reasoning, and dig further into the reasoning behind another's strategy. A question that dependably prompts students to describe how representations connect is: "Where do you see the __ in ___?" In the previous example, that might sound like, "Where do you see the <u>number of bricks in the arch</u> in <u>the bar model</u>?" It's important to pause students' explanations occasionally to ask classmates to explain and process what has been said.

4 Pause during Key Points in Student Explanations to Allow Students to Process Ideas

Check for understanding and maintain engagement and perseverance by stopping occasionally to ask classmates to restate or rephrase what the explainer has shared. Pause student explanations at critical points to ask classmates questions about the students' explanation. Questions that can be used for any lesson include, "Why do you think the student did that?" or "What do you think should happen next?" These strategies allow students to hear an idea multiple times in different ways so they can internalize what they are hearing with greater understanding. It also engages multiple students in the classroom conversation.

5 Ask Questions That Help Students Reflect on the Process

Developing students' capacity to reflect on their own thinking and that of others is a helpful way to articulate and solidify thinking that can be applied again in a new situation. Having students process which strategies were most effective or understandable will help them when they encounter similar problems in the future.

Making Productive Struggle Manageable in the Classroom

While supporting productive struggle may seem overwhelming, it's more manageable with the use of routines and lesson frameworks. The Try–Discuss–Connect instructional framework found in *i-Ready Classroom Mathematics* (see Figure 12) provides a cohesive and repeatable way for teachers to integrate teaching strategies that support productive struggle at each critical step of the problem-solving process.

Try–Discuss–Connect Framework

The framework begins with the **Try (i.e., Steps 1 and 2)**, during which the class works together using the Three Reads and other routines to make sense of a problem together. Students are then given a short amount of private think time to begin working on the problem of the lesson. Questions in the support materials provide teachers with questions they might ask students to support productive struggle.

Students then **Discuss (i.e., Steps 3 and 4)**, first with a partner and then with the whole class. They work together with a partner to continue to solve the problem and discuss their thinking using the provided sentence frames to guide partner conversations. Teachers facilitate a whole class discussion of multiple selected student strategies and text strategies, having students rephrase or answer questions at key points in the explanations to engage the whole class and check for understanding.

Students then **Connect (i.e., Steps 5 and 6)** classroom strategies, reflect on which strategies they might use for future problems, and then apply their thinking to new problems.

The steps embedded in the Try–Discuss–Connect framework support students in the three key areas where they naturally fall out of productive struggle (see page 10). The strategies used are key to developing students' ability to productively struggle and develop more fully into mathematical thinkers.

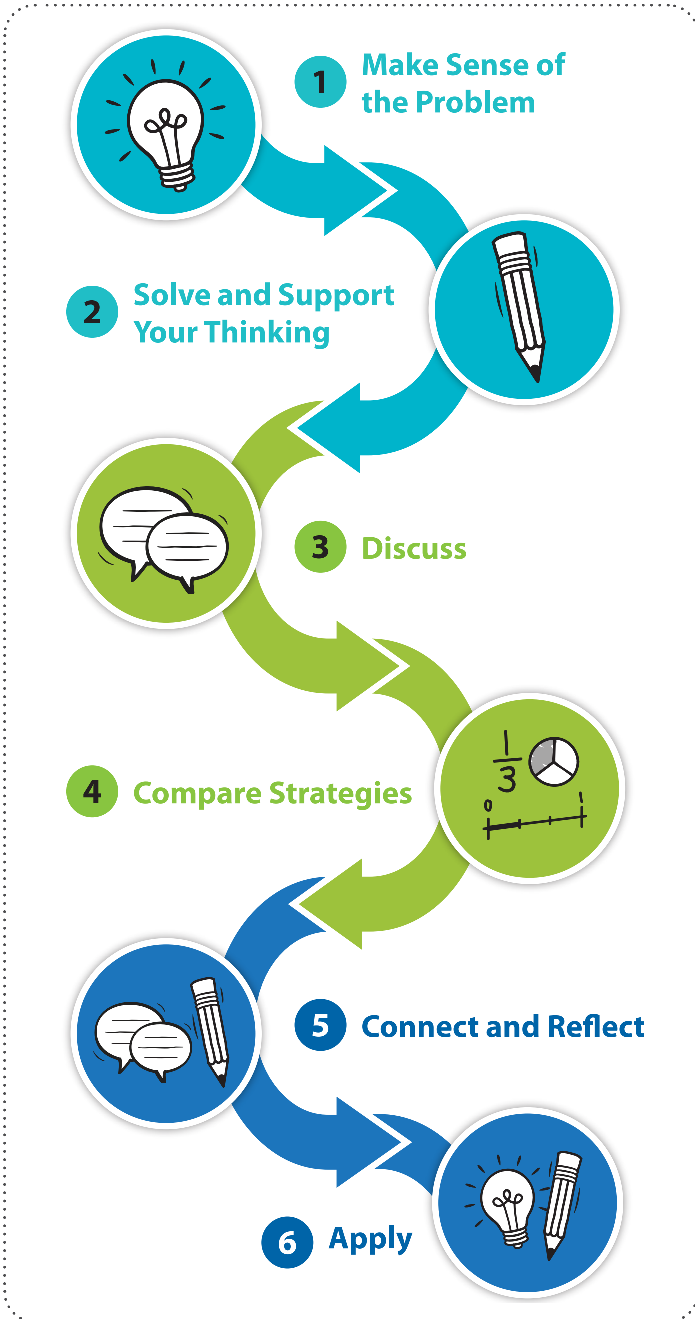


Figure 12.
The Try–Discuss–Connect instructional framework from *i-Ready Classroom Mathematics*

The Try–Discuss–Connect framework will support you as you:

- Give students a starting point by helping them develop a process to make sense of the problem. This includes identifying the quantities and relationships in the problems and using the Three Reads routine and notecatcher to make sense of problems.
- Give students time to think about the problem on their own.
- Guide students who are stuck or who are pursuing a non-productive solution by asking students questions to think about—and then walking away.
- Provide students time to talk with a partner about each other's strategies. Use sentence frames to help students get started and focus their conversation on the mathematics.
- Have a few selected students share different strategies based on the goals of the lesson. As students explain their strategies, pause them at key points in the explanation to check for understanding and engage classmates in processing what is being said.
- Guide students to make connections between strategies by asking them what is the same and what is different, as well as looking at how different strategies represent the quantities and relationships in the problem.
- Ask students to reflect on the process and apply their thinking to new problems.

Conclusion

For many of us, it's uncomfortable to allow students to struggle, even if they are doing so productively. As students work through a high-demand, non-routine task and are close to breaking through to a solution strategy, we may be tempted to point out the one missing piece that would set students on a faster track to an answer. Doing so, however, robs students of the opportunity to continue in their productive struggle, and it sends the message that their thinking wasn't moving along quickly enough or that they would have never found the missing piece on their own.

There are times when we want so badly for our students to feel good about themselves that we "over-steer" students and offer suggestions or reminders that lower the cognitive demand. This removes the productive struggle, albeit with good intentions.

Productive struggle—for both teachers and students—requires a shift in thinking about the purpose of a mathematics classroom. While many of us learned in classrooms with teachers at the front telling us what to do for every step of a given problem, we now know that learning gains are greater in classrooms that look different, where students are experiencing productive struggle on a regular basis. It takes time to develop this type of classroom, but in the end, supporting productive struggle is all about teaching students that they are absolutely able to do meaningful mathematics when given the time and space to think.

To get started:

- Select one of the four key elements highlighted on pages 8–9.
- Choose one of the three critical points, introduced on page 10, to focus on as you begin to support productive struggle within a high-demand lesson.
- Regularly reflect on the four key elements and teacher tips so that over time you are building myriad tools for you and your students.
- Be forgiving. Supporting productive struggle is complex work that requires consistent effort.

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