

Curriculum Associates RESEARCH

Predicting Algebra Readiness

Mathematics

Jeffrey Yo, M.A. and Jennifer Sallman, Ph.D.

Research Report, January 2024

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Summary

Algebra readiness plays a critical role in shaping students' academic and life trajectories. The purpose of this study was to understand how the combined and unique contributions of previous Mathematics domain performance are related to future overall mathematics performance, specifically algebra readiness. Student mathematics performance was tracked using data from the *i-Ready Diagnostic* between the 2020–2021 through 2022–2023 school years for five cohorts of students in Grades 2–6.

Overall, this study found that for all cohorts, Mathematics domain placements in Year 1 (i.e., winter 2021) of the study were strongly related to overall mathematics scores in Year 3 (i.e., spring 2023). Further, we found that all domains are important for future algebra success, suggesting a holistic approach to teaching mathematics.

While prior performance was predictive of future performance, in all cohorts, we also found the percentage of students who ended Year 3 on track or algebra ready is higher than the predicted percentage. In other words, this study shows that students' starting placement does not predetermine their end placement, especially for the earlier grades. While we do not know what students' experiences were between Year 1 and Year 3, these trends suggest that using previous performance to provide students with the appropriate, targeted, and effective instruction can perhaps accelerate students' mathematics trajectories toward algebra readiness.

Introduction

Researchers and educators have widely acknowledged the crucial role of algebra readiness in shaping students' life trajectories, with success in algebra correlated to graduating high school, attending college, and securing future employment (ACT, 2006; NMAP, 2008). Studies indicate that algebra readiness is associated with higher academic achievement, increased enrollment in advanced mathematics courses, and higher rates of college attendance (NCES, 2001; Spielhagen, 2006). Yet, supporting all students to be algebra ready by high school remains a concern as mathematics scores among US school children continue to decline—a trend exacerbated by the COVID-19 pandemic (Curriculum Associates, 2023b; NCES, 2022). As a result, educators need insights into accelerating student learning to enhance their students' algebra readiness, ultimately contributing to children's long-term success.

The purpose of this study was to understand how the combined and unique contributions of previous Mathematics domain performance is related to later overall mathematics performance. The *i-Ready Diagnostic* for Mathematics provides domain-level data for four domains: Number and Operations, Algebra and Algebraic Thinking, Measurement and Data, and Geometry.

Student mathematics performance was tracked using data from the Diagnostic over two years for five cohorts of students in Grades 2–6. Using linear regression and supplemented by a descriptive exploration of student placement patterns, this longitudinal analysis found that Mathematics domain placements in Year 1 of the study were predictive of overall mathematics scores two years later across cohorts. Ultimately, all domains are important for future algebra success, suggesting a comprehensive approach to teaching mathematics.

Methodology

Research Questions

This study was designed to address the following research question:

1. How does the domain-level performance in mathematics in Grades 2, 3, 4, 5, and 6 predict overall mathematics performance two years later in Grades 4, 5, 6, 7, and 8, respectively?

Sample

Students who were in Grades 2–6 during the 2020–2021 school year were eligible for inclusion in this study. To be included in the analysis, students had to complete an *i-Ready Diagnostic* for Mathematics in winter and spring of the 2020–2021 school year as well as two years later during spring of the 2021–2022 and 2022–2023 school years. Although this study utilized assessment taken in winter 2020–2021 (i.e., Year 1) and spring 2022–2023 (i.e., Year 3), we

required that students completed assessments in additional time points for comparability of findings in future longitudinal analyses.

Students were excluded from the final sample if: 1) their chronological grade level at any point in time did not match the expected grade level (e.g., if the student was retained in a grade), 2) the student’s Diagnostic was flagged with a red Rush flag, indicating that the student spent so little time on the assessment that they were likely “rushing” through the assessment with little effort, or 3) the student self-reported not taking the Diagnostic in person¹. There is one exception for Grade 3 students for whom the self-reported testing location data was unavailable.

Table 1 shows students’ grade level at the beginning and end of the study and the final sample size for each grade-level cohort. As school districts are not required to report demographic information for their students to Curriculum Associates, reliable demographic data about this sample was not available.

Table 1. Cohorts by Grade Level and Sample Size

Name of Cohort	Year 1 Winter → Year 3 Spring	N Sample
Grade 2 Cohort	Grade 2 → Grade 4	179,341
Grade 3 Cohort	Grade 3 → Grade 5	397,591
Grade 4 Cohort	Grade 4 → Grade 6	126,806
Grade 5 Cohort	Grade 5 → Grade 7	97,562
Grade 6 Cohort	Grade 6 → Grade 8	69,787

Note: As the data did not have the test location for Grade 3 in the 2020–2021 school year, the Year 1 winter score for the Grade 3 cohort includes all students, regardless of testing location.

Diagnostic Placement Levels

The Diagnostic classifies students into criterion-referenced placement levels based on a scale score for both overall mathematics achievement and domain achievement (Curriculum Associates, 2023a). For the purposes of the exploratory analyses, students were placed into risk categories. See [Table 2](#) for a crosswalk between the Diagnostic’s criterion-referenced placement levels and the categories used in this analysis.

¹Starting in fall 2020, we asked students if they were taking the Diagnostic in the school building at the start of each testing session.
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Table 2. Crosswalk of Analysis Categories and Diagnostic Grade-Level Placements

Analysis Category	Grade-Level Placement
Two or More Grade Levels Below	Two Grade Levels Below Three or More Grade Levels Below
One Grade Level Below	One Grade Level Below
Early On Grade Level	Early On Grade Level
Mid or Above Grade Level	Mid On Grade Level Late On Grade Level Above Grade Level

Algebra Ready Performance-Level Standards

Two performance-level standards were used to classify students as “algebra ready.” Students were classified as “on track” to being algebra ready by the end of Grade 8 if they end the year with a Mid or Above Grade Level placement for their chronological grade. A student is classified as algebra ready if they meet or exceed a score of 541 on the Diagnostic for Mathematics in Grades 5 and above, which represents a Mid On Grade Level placement for a Grade 8 student. See Table 3 for cohort-specific placement-level standards.

Table 3. Algebra Ready Performance-Level Standards by Cohort

Name of Cohort	Grade in Spring 2023	On Track or Algebra Ready Spring 2023 Mathematics Performance-Level Standard
Grade 2 Cohort	4	482
Grade 3 Cohort	5	498
Grade 4 Cohort	6	514
Grade 5 Cohort	7	531
Grade 6 Cohort	8	541

We used the performance-level standards reflected in Table 3 to maintain a high bar of rigor for these analyses. However, we could have selected a more inclusive performance-level standard that considers the standard error of measurement (SEM). More specifically, a more inclusive performance-level standard of algebra ready is one minimum SEM below 541, or a score of 535. While we do not use this measure in our analyses, we discuss how using a more inclusive measure would have impacted our results in the discussion section.

Furthermore, we feel confident in using the performance-level standard of Mid or Above Grade Level placement for a student’s chronological grade as a strong proxy for being on track for algebra readiness by the end of Grade 8. We have found that the vast majority of students who place Mid or Above Grade Level in prior years in fact meet the algebra ready performance-level standard in future years. For example, in the Grade 4 cohort, we found that 92% of those who

placed Mid or Above Grade Level in Year 1 continued to be Mid or Above Grade Level in Year 2, with 14% already algebra ready by the end of Grade 5. We see this pattern continue into Year 3 where 87% of those who placed Mid or Above Grade Level in Year 1 continue to be Mid or Above Grade Level in Year 3, with 50% algebra ready by the end of Grade 6. We find a similar pattern in the Grade 5 cohort. More specifically, 85% of those who place Mid or Above Grade Level in Year 1 also place Mid or Above Grade Level in Year 2, with 44% algebra ready by the end of Grade 6, and 77% who place Mid or Above Grade Level in Year 1 also place Mid or Above Grade Level in Year 3, with 62% already algebra ready by the end of Grade 7. For more placement-level details, see Tables 6 and 7 below.

Analyses

Spring overall mathematics scale scores during the 2022–2023 school year were predicted using winter domain placements from the 2020–2021 school year using linear regression. Linear regression was chosen as the analysis method because it allows the use of all domains simultaneously to predict the outcome. Within the construct of mathematics, each domain is related to other domains, so regression allowed us to isolate the unique contribution of a single domain over and above the contribution of other domains.

For each cohort, the following model was fitted:

$$Y_i = \beta_0 + \sum\beta_1(NO) + \sum\beta_2(AL) + \sum\beta_3(MS) + \sum\beta_4(GEO)$$

In this equation, Y_i represents the predicted overall mathematics score in Year 3 for student i , which is predicted as a function of the intercept (i.e., the predicted Year 3 score when the Year 1 placement for all domains is Mid or Above Grade Level, represented by β_0) and the point estimate associated with each of the student's domain placements. The terms $\sum\beta_1$ through $\sum\beta_4$ each represent a vector of dummy-coded point estimates for the possible placements in that domain for the relevant cohort. Each vector excluded the placement associated with Mid or Above Grade Level expectations because it was used as the reference. Therefore, the point estimate for the Mid or Above Grade Level placement was zero. As such, the intercept can be interpreted as the estimated Year 3 mathematics score for a student who scored Mid or Above Grade Level in all domains assessed in Year 1. The estimated Year 3 mathematics score for a student whose domain-specific placement in Year 1 was anything other than Mid or Above Grade Level can be calculated by adding the point estimate associated with that domain-specific placement to the intercept. The models also allow the calculation of R^2 , the squared multiple correlation, which provides a measure of the proportion of variance in the outcome that is explained by the predictors (Pedhazur, 1997).

All analyses were conducted in R version 4.1.3 (R Core Team, 2022). Visual inspections of the models and data confirmed there were no major causes for concern about violation of the assumptions of linear regression (i.e., linearity, normality, and homoscedasticity of residuals). Assumptions were also not violated regarding multicollinearity or outlier effects.

Results

Overall, this analysis found that for all cohorts, Mathematics domain placements in Year 1 of the study were strongly related to overall mathematics scores two years later. More specifically, we found that all domains are important for future algebra success suggesting a holistic approach to teaching mathematics. While all domains are important, we found that Algebraic Thinking for all cohorts and Algebraic Thinking and Number and Operations for Grades 5 and 6 cohorts were slightly more related to overall future mathematics performance than the Geometry or Measurement and Data domains. As such, when prioritizing mathematics instruction to accelerate student learning, we do not suggest deprioritizing the Algebraic Thinking or Number and Operations domains.

Examining Observed Placement Levels

As a preliminary analysis, descriptive patterns of domain placements in Year 1 compared to overall mathematics placements in Year 3 were examined. For these tables, placements were grouped into five categories: Three or More Grade Levels Below, Two Grade Levels Below, One Grade Level Below, Early On Grade Level, and Mid or Above Grade Level. These different categorizations are based on instructional “views” available in the platform for educators. Across cohorts and domains, the vast majority of students who start Year 1 Mid Above Grade Level will end Mid Above Grade Level, which means the majority of those students will be on track or algebra ready. Similarly, the vast majority of students who start Year 1 Early On Grade Level will end Year 3 either Mid Above or Early On Grade Level. In cohorts 2 and 3, if a student started Year 1 One Grade Level Below, more than half would end Year 3 Early On or Mid Above Grade Level. However, for cohorts 4–6, if a student starts Year 1 One Grade Level Below, more than 50% will remain one or more grade levels below in Year 3. Similarly, for cohorts 2 and 3, if a student started Year 1 Two Grade Levels Below, about half of those students would end Year 3 Two or More Grade Levels Below. However, for cohorts 4–6, if a student starts Year 1 Two Grade Levels Below, the majority will end Year 3 Two or More Grade Levels Below. Lastly, for cohorts 3–6, the majority of students who start Year 1 Three or More Grade Levels Below will remain Three or More Grade Levels Below in Year 3. In other words, students who start at higher placement levels tend to end at higher placement levels and vice versa.

Across cohorts and domains, a greater proportion of students were considered on track to being algebra ready in Year 3 if they placed higher on the Year 1 Diagnostic. For instance, in the Grade 2 cohort, 38% of students are on track to be algebra ready by the end of Grade 4. Of the Grade 2 students with an overall Mid or Above Grade Level placement, 90% were on track to being algebra ready (i.e., placed Mid or Above Grade Level) by the end of Grade 4. The proportion of on-track students decreases with lower placement levels, as 69%, 25%, and 2% of students are considered on track in Grade 4 when they are placed overall in Early On Grade Level, One Grade Level Below, and Two Grade Levels Below, respectively. Table 4 includes results for the Grade 2 cohort for each domain.

Table 4. Grade 2 Placement by Domain in Year 1 and Placement by Overall Mathematics in Grade 4 (Year 3)

Year 1 (Winter 2021)				Year 3 (Spring 2023) Overall Mathematics Placement Distribution				
Cohort	Domain	Domain Placement	Number of Students	Mid or Above Grade Level	Early On Grade Level	One Grade Level Below	Two Grade Levels Below	Three or More Grade Levels Below
2 → 4	Overall Placement	Mid or Above Grade Level	24,005	90%	8%	2%	.2%	.1%
		Early On Grade Level	29,947	69%	24%	6%	.3%	.1%
		One Grade Level Below	100,451	25%	34%	34%	5%	2%
		Two Grade Levels Below	24,938	2%	10%	38%	25%	25%
	Number and Operations	Mid or Above Grade Level	30,304	78%	16%	5%	1%	.2%
		Early On Grade Level	40,378	57%	29%	13%	1%	.2%
		One Grade Level Below	85,125	24%	32%	36%	7%	2%
		Two Grade Levels Below	23,534	3%	11%	38%	24%	25%
	Algebra and Algebraic Thinking	Mid or Above Grade Level	32,118	80%	14%	5%	1%	.2%
		Early On Grade Level	40,218	57%	29%	13%	1%	.4%
		One Grade Level Below	91,060	21%	32%	38%	8%	3%
		Two Grade Levels Below	15,945	2%	8%	32%	26%	32%
	Measurement and Data	Mid or Above Grade Level	38,480	77%	17%	6%	1%	.2%
		Early On Grade Level	24,854	55%	29%	14%	2%	1%
		One Grade Level Below	85,338	26%	32%	33%	6%	3%
		Two Grade Levels Below	30,669	7%	17%	40%	18%	18%
	Geometry	Mid or Above Grade Level	47,381	73%	19%	7%	1%	.2%
		Early On Grade Level	23,071	53%	31%	15%	1%	1%
		One Grade Level Below	70,922	26%	32%	34%	6%	3%
		Two Grade Levels Below	37,967	7%	19%	41%	17%	15%

Note: For Grade K, there is no placement level that is Two or More Grade Levels Below. Furthermore, as there are no Grade 9 Algebra items in the Grade 4 *i-Ready Diagnostic*, this Diagnostic cannot detect who is considered algebra ready in Grade 4.

In the Grade 3 cohort, 3% are algebra ready² and 31% of students are on track to being algebra ready by the end of Grade 5. Of the Grade 3 students with an overall Mid or Above Grade Level placement, 19% were algebra ready and an additional 63% were on track to being algebra ready by Year 3. These proportions were smaller among students who had an overall Early On Grade Level placement in Grade 3, as 2% were considered algebra ready and an additional 62% were on track in Year 3. The proportion of on-track students decreases with lower placement levels, as 22%, 2%, and 1% of students are considered on track in Grade 5 when they are placed overall One Grade Level Below, Two Grade Levels Below, and Three or More Grade Levels Below, respectively. [Table 5](#) includes results for the Grade 3 cohort for each domain.

²In the majority of cases, students in Grades K–6 will benefit most from traditional mathematics instruction, and while students may be categorized as algebra ready, we do not recommend algebra-specific courses for students in chronological Grades K–6.

Table 5. Grade 3 Placement by Domain in Year 1 and Placement by Overall Mathematics in Grade 5 (Year 3)

Year 1 (Winter 2021)				Year 3 (Spring 2023) Overall Mathematics Placement Distribution					
Cohort	Domain	Domain Placement	Number of Students	Algebra Ready	Mid or Above Grade Level	Early On Grade Level	One Grade Level Below	Two Grade Levels Below	Three or More Grade Levels Below
3 → 5	Overall Placement	Mid or Above Grade Level	47,836	19%	82%	9%	6%	2%	1%
		Early On Grade Level	76,506	2%	64%	23%	10%	1%	1%
		One Grade Level Below	192,781	.2%	23%	30%	37%	7%	3%
		Two Grade Levels Below	56,148	0%	2%	8%	40%	26%	24%
		Three or More Grade Levels Below	24,320	0%	1%	3%	16%	19%	62%
	Number and Operations	Mid or Above Grade Level	67,253	13%	66%	17%	13%	3%	2%
		Early On Grade Level	70,781	2%	51%	26%	19%	3%	2%
		One Grade Level Below	198,868	1%	26%	26%	35%	9%	5%
		Two Grade Levels Below	37,974	0%	4%	9%	36%	24%	27%
		Three or More Grade Levels Below	22,715	0%	1%	3%	17%	19%	61%
	Algebra and Algebraic Thinking	Mid or Above Grade Level	105,589	10%	71%	17%	9%	2%	1%
		Early On Grade Level	74,766	1%	45%	31%	21%	3%	1%
		One Grade Level Below	155,176	.1%	16%	26%	41%	11%	6%
		Two Grade Levels Below	46,431	0%	2%	7%	36%	25%	30%
		Three or More Grade Levels Below	15,629	0%	1%	2%	14%	17%	66%
	Measurement and Data	Mid or Above Grade Level	98,999	10%	69%	18%	10%	2%	1%
		Early On Grade Level	53,446	2%	48%	29%	19%	2%	1%
		One Grade Level Below	152,641	.4%	23%	28%	36%	8%	5%
		Two Grade Levels Below	59,544	.1%	5%	13%	41%	21%	21%
		Three or More Grade Levels Below	32,961	0%	2%	5%	25%	21%	46%
Geometry	Mid or Above Grade Level	63,001	12%	72%	16%	9%	2%	1%	
	Early On Grade Level	47,421	4%	58%	23%	15%	2%	1%	
	One Grade Level Below	196,152	1%	29%	27%	33%	7%	4%	
	Two Grade Levels Below	52,237	.1%	7%	15%	40%	19%	19%	
	Three or More Grade Levels Below	38,780	0%	2%	6%	26%	22%	44%	

Note: The Year 1 winter score for the Grade 3 cohort includes all students, regardless of testing location.

In the Grade 4 cohort, 17% of students are on track to being algebra ready by the end of Grade 6, and an additional 9% of students are algebra ready. For students with an overall Mid or Above Grade Level placement, 50% were algebra ready and an additional 37% were considered on track. Among students who had an overall Early On Grade Level placement, 13% were

considered algebra ready and an additional 37% were on track. For students who placed One Grade Level Below, 1% and 10% were considered algebra ready and on track, respectively. The proportion of on-track students decreases with lower placement levels, as 1%, and .2% of students are considered on track in Grade 6 when they are placed overall Two Grade Levels Below, and Three or More Grade Levels Below, respectively. Table 6 includes results for the Grade 4 cohort for each domain.

Table 6. Grade 4 Placement by Domain in Year 1 and Placement by Overall Mathematics in Grade 6 (Year 3)

Year 1 (Winter 2021)				Year 3 (Spring 2023) Overall Mathematics Placement Distribution					
Cohort	Domain	Domain Placement	Number of Students	Algebra Ready	Mid or Above Grade Level	Early On Grade Level	One Grade Level Below	Two Grade Levels Below	Three or More Grade Levels Below
4 → 6	Overall Placement	Mid or Above Grade Level	15,293	50%	87%	11%	2%	.2%	.2%
		Early On Grade Level	28,026	13%	50%	39%	10%	1%	.4%
		One Grade Level Below	57,204	1%	11%	35%	41%	8%	5%
		Two Grade Levels Below	16,300	.1%	1%	6%	36%	25%	32%
		Three or More Grade Levels Below	9,983	.1%	.3%	2%	11%	14%	73%
	Number and Operations	Mid or Above Grade Level	27,039	32%	67%	24%	8%	1%	1%
		Early On Grade Level	35,124	7%	31%	38%	25%	4%	2%
		One Grade Level Below	43,001	2%	11%	29%	39%	12%	10%
		Two Grade Levels Below	14,509	.4%	2%	11%	33%	21%	33%
		Three or More Grade Levels Below	7,133	.1%	.4%	2%	11%	12%	75%
	Algebra and Algebraic Thinking	Mid or Above Grade Level	36,581	28%	64%	27%	8%	1%	1%
		Early On Grade Level	27,459	5%	27%	43%	26%	3%	2%
		One Grade Level Below	39,589	1%	7%	28%	44%	12%	9%
		Two Grade Levels Below	14,508	.2%	1%	7%	34%	24%	35%
		Three or More Grade Levels Below	8,669	.1%	.4%	2%	12%	14%	72%
	Measurement and Data	Mid or Above Grade Level	30,218	29%	64%	26%	9%	1%	1%
		Early On Grade Level	22,711	9%	34%	40%	22%	3%	2%
		One Grade Level Below	45,131	3%	14%	32%	38%	10%	7%
		Two Grade Levels Below	15,887	1%	3%	13%	38%	20%	25%
		Three or More Grade Levels Below	12,859	.1%	1%	4%	19%	17%	59%
Geometry	Mid or Above Grade Level	11,533	45%	78%	17%	5%	1%	.4%	
	Early On Grade Level	21,671	17%	49%	33%	15%	2%	1%	
	One Grade Level Below	50,786	6%	24%	35%	31%	6%	5%	
	Two Grade Levels Below	29,274	1%	7%	21%	38%	16%	18%	
	Three or More Grade Levels Below	13,542	.2%	2%	6%	21%	16%	55%	

For the Grade 5 cohort, 15% of students are algebra ready by the end of Grade 7, with an additional 7% on track. For students with an overall Mid or Above Grade Level placement, 62% were algebra ready, and an additional 15% were on track. For those with an overall Early On Grade Level placement, 19% were algebra ready and an additional 14% were on track. Students placed One Grade Level Below had 3% who were algebra ready and 3% on track, while those

placed Two Grade Levels Below had 1% in both categories. Less than one percent of the students who were Three or More Grade Levels Below were algebra ready or on track. Table 7 includes results for the Grade 5 cohort for each domain.

Table 7. Grade 5 Placement by Domain in Year 1 and Placement by Overall Mathematics in Grade 7 (Year 3)

Year 1 (Winter 2021)				Year 3 (Spring 2023) Overall Mathematics Placement Distribution					
Cohort	Domain	Domain Placement	Number of Students	Algebra Ready	Mid or Above Grade Level	Early On Grade Level	One Grade Level Below	Two Grade Levels Below	Three or More Grade Levels Below
Grades 5 → 7	Overall Placement	Mid or Above Grade Level	14,422	62%	77%	20%	3%	.3%	.2%
		Early On Grade Level	22,034	19%	33%	44%	20%	2%	1%
		One Grade Level Below	38,449	3%	7%	26%	46%	12%	10%
		Two Grade Levels Below	11,722	1%	1%	5%	31%	24%	38%
		Three or More Grade Levels Below	10,935	.3%	.4%	1%	8%	11%	79%
	Number and Operations	Mid or Above Grade Level	23,002	45%	59%	29%	10%	1%	1%
		Early On Grade Level	19,673	14%	24%	40%	29%	4%	3%
		One Grade Level Below	41,357	3%	6%	20%	41%	15%	18%
		Two Grade Levels Below	5,202	1%	1%	6%	25%	20%	49%
		Three or More Grade Levels Below	8,328	.3%	1%	2%	10%	10%	77%
	Algebra and Algebraic Thinking	Mid or Above Grade Level	15,779	50%	64%	25%	9%	1%	1%
		Early On Grade Level	22,875	20%	32%	38%	24%	3%	3%
		One Grade Level Below	38,871	5%	9%	26%	41%	13%	12%
		Two Grade Levels Below	9,511	1%	1%	6%	31%	22%	40%
		Three or More Grade Levels Below	10,526	.3%	1%	1%	10%	12%	76%
	Measurement and Data	Mid or Above Grade Level	27,281	41%	55%	31%	12%	1%	1%
		Early On Grade Level	16,223	12%	21%	37%	33%	6%	4%
		One Grade Level Below	31,418	4%	8%	24%	41%	13%	13%
		Two Grade Levels Below	9,694	1%	2%	10%	33%	21%	34%
		Three or More Grade Levels Below	12,946	1%	1%	3%	15%	14%	67%
Geometry	Mid or Above Grade Level	13,723	50%	64%	26%	9%	1%	1%	
	Early On Grade Level	17,648	24%	36%	36%	23%	3%	2%	
	One Grade Level Below	35,271	8%	14%	30%	37%	10%	9%	
	Two Grade Levels Below	12,895	2%	4%	15%	39%	18%	24%	
	Three or More Grade Levels Below	18,025	1%	2%	6%	21%	15%	57%	

For the Grade 6 cohort, 21% of students finish Grade 8 algebra ready. Eighty-one percent, 37%, 8%, 2%, and 1% of students with an overall placement of Mid or Above Grade Level, Early On Grade Level, One Grade Level Below, Two Grade Levels Below, and Three or More Grade Levels Below in Grade 6 end Grade 8 algebra ready. [Table 8](#) includes results for the Grade 6 cohort for each domain.

Table 8. Grade 6 Placement by Domain in Year 1 and Placement by Overall Mathematics in Grade 8 (Year 3)

Year 1 (Winter 2021)				Year 3 (Spring 2023) Overall Mathematics Placement Distribution				
Cohort	Domain	Domain Placement	Number of Students	Mid or Above Grade Level	Early On Grade Level	One Grade Level Below	Two Grade Levels Below	Three or More Grade Levels Below
Grades 6 → 8	Overall Placement	Mid or Above Grade Level	8,133	81%	15%	3%	.3%	.3%
		Early On Grade Level	16,557	37%	37%	23%	2%	1%
		One Grade Level Below	24,840	8%	22%	46%	13%	12%
		Two Grade Levels Below	8,671	2%	6%	31%	20%	41%
		Three or More Grade Levels Below	11,586	1%	2%	9%	10%	78%
	Number and Operations	Mid or Above Grade Level	14,192	62%	24%	12%	1%	1%
		Early On Grade Level	12,778	28%	33%	31%	5%	4%
		One Grade Level Below	24,759	9%	21%	41%	13%	16%
		Two Grade Levels Below	11,112	2%	5%	26%	17%	50%
		Three or More Grade Levels Below	6,946	1%	1%	8%	7%	83%
	Algebra and Algebraic Thinking	Mid or Above Grade Level	10,466	67%	21%	10%	1%	1%
		Early On Grade Level	14,276	33%	33%	27%	4%	3%
		One Grade Level Below	25,134	11%	23%	41%	12%	14%
		Two Grade Levels Below	9,104	3%	8%	33%	18%	39%
		Three or More Grade Levels Below	10,807	1%	2%	11%	10%	76%
	Measurement and Data	Mid or Above Grade Level	18,262	55%	27%	15%	2%	1%
		Early On Grade Level	10,938	25%	32%	34%	5%	5%
		One Grade Level Below	21,148	9%	19%	41%	14%	17%
		Two Grade Levels Below	7,242	3%	9%	32%	18%	39%
		Three or More Grade Levels Below	12,197	2%	3%	14%	11%	71%
Geometry	Mid or Above Grade Level	8,383	64%	22%	12%	1%	1%	
	Early On Grade Level	14,190	37%	31%	26%	4%	3%	
	One Grade Level Below	23,882	15%	24%	38%	11%	12%	
	Two Grade Levels Below	8,723	5%	11%	35%	16%	33%	
	Three or More Grade Levels Below	14,609	2%	4%	17%	12%	65%	

These tables provide preliminary evidence for the predicted pattern of relationships between students’ performance in early Mathematics domains and subsequent overall mathematics achievement. In general, the vast majority of students who started Year 1 Mid or Above Grade Level are considered algebra ready or on track in Year 3. This pattern is consistent across cohorts, but as the grade of the cohort increases, the overall proportion of students on track to

being algebra ready decreases. Unfortunately, this pattern of fewer students placing on grade level as chronological grade increases mirrors general trends seen in previous research (Curriculum Associates, 2023b).

Predicting Overall Mathematics Scale Scores

To examine the relationship of individual domains with later overall mathematics performance, we used linear regression to predict overall mathematics scale scores in Year 3 from the Year 1 domain placement levels for each cohort. Across all cohorts, lower placement levels within each domain in Year 1 of the study predicted a lower overall mathematics score in Year 3 of the study, while higher placement levels predicted higher overall mathematics scores. While all domains contribute to future algebra success across cohorts, there were differences by cohort and domain in the magnitude of the point estimates and in the corresponding overall mathematics score prediction. Algebraic Thinking shows a consistently stronger predictive relationship across all cohorts. Specifically, for Grades 5 and 6 students, Algebraic Thinking and Number and Operations exhibit a stronger predictive relationship compared to the Measurement and Data and Geometry domains. Detailed results of the regression analyses for each cohort are provided in the tables in [Appendix B](#).

In the Grade 2 cohort, students who placed Mid or Above Grade Level in all four domains in winter of Year 1 were predicted to have an overall mathematics score of 510 by the end of Year 3 (i.e., spring of Grade 4). As such, a score of 510 is the intercept and can be interpreted as the baseline for the Grade 2 cohort in this model (see Table 9). For Grade 2 students who placed Early On Grade Level in all domains, the predicted overall score in Grade 4 was 487, or 23 scale score points lower than the baseline.

If a Grade 2 student is Mid or Above Grade Level or Early On Grade Level in all domains in winter 2021, their predicted Grade 4 spring 2023 mathematics score (e.g., 487 or higher) would be higher than 482, exceeding the performance-level standard for being considered on track for algebra readiness in Grade 4. Overall, 34% of students are predicted to be on track. However, if a Grade 2 student is Early On Grade Level in all three domains but One Grade Level Below in either Number and Operations or Algebra, their predicted Grade 4 spring 2023 mathematics score is not on track for algebra readiness. The model's R^2 was .55, indicating that the model explained about 55% of the variance in Grade 4 scores.

Table 9. Predicting Grade 4 Overall Mathematics Score Based on Grade 2 Mathematics Domain Placements

Winter 2021 Domain Combinations					
Overall Combination	Mid or Above Grade Level	Early On Grade Level	One Grade Level Below	Predicted Overall Mathematics Score in Grade 4	Difference from Baseline
All Domains Mid or Above Grade Level	NO + AL + MS + GEO			510	0
All Domains Early On Grade Level		NO + AL + MS + GEO		487	-23
Three Early On Grade Level Domains and a One Grade Level Below Domain		NO + AL + MS	GEO	483	-27
		NO + AL + GEO	MS	482	-28
		AL + MS + GEO	NO	481	-29
		NO + MS + GEO	AL	479	-31
All Domains One Grade Level Below			NO + AL + MS + GEO	463	-47

Note: NO, AL, MS, and GEO refer to Number and Operations, Algebra and Algebraic Thinking, Measurement and Data, and Geometry, respectively. The difference from baseline when multiple domains are added may differ slightly from the sum of the differences reported for individual domains due to rounding. More precise point estimates and all combinations of domains can be found in [Appendix B](#).

Table 10 includes results for the Grade 3 cohort. In the Grade 3 cohort, students who placed Mid or Above Grade Level in all four domains in winter of Year 1 were predicted to have an overall mathematics score of 521 by the end of Year 3 (i.e., spring of Grade 5). As such, a score of 521 is the intercept and can be interpreted as the baseline for the Grade 3 cohort in this model. For Grade 3 students who placed Early On Grade Level in all domains, the predicted overall score in Grade 5 was 500, or 21 scale score points lower than the baseline.

If a Grade 3 student is Mid or Above Grade Level or Early On Grade Level in all domains in winter 2021, their predicted Grade 5 spring 2023 mathematics score (e.g., 500 or higher) would be higher than 498, exceeding the performance-level standard for being considered on track for algebra readiness in Grade 5. Overall, 28% of students are predicted to be on track. However, if a Grade 3 student is Early On Grade Level in all three domains but One Grade Level Below in any one domain, their predicted Grade 5 spring 2023 mathematics score is not on track for algebra readiness. The model’s R^2 was .54, indicating that the model explained about 54% of the variance in Grade 5 scores.

Table 10. Predicting Grade 5 Overall Mathematics Score Based on Grade 3 Mathematics Domain Placements

Winter 2021 Domain Combinations					
Overall Combination	Mid or Above Grade Level	Early On Grade Level	One Grade Level Below	Predicted Overall Mathematics Score in Grade 5	Difference from Baseline
All Domains Mid or Above Grade Level	NO + AL + MS + GEO			521	
All Domains Early On Grade Level		NO + AL + MS + GEO		500	-21
Three Early On Grade Level Domains and a One Grade Level Below Domain		AL + MS + GEO	NO	496	-25
		NO + AL + MS	GEO	495	-26
		NO + AL + GEO	MS	493	-28
		NO + MS + GEO	AL	490	-31
All Domains One Grade Level Below			NO + AL + MS + GEO	475	-46

Note: NO, AL, MS, and GEO refer to Number and Operations, Algebra and Algebraic Thinking, Measurement and Data, and Geometry domains, respectively. The difference from baseline when multiple domains are added may differ slightly from the sum of the differences reported for individual domains due to rounding. More precise point estimates and all combinations of domains can be found in [Appendix B](#).

As Table 11 shows, students who placed Mid or Above Grade Level in all four domains in winter of Year 1 were predicted to have an overall mathematics score of 542 by the end of Year 3 (i.e., spring of Grade 6). As such, a score of 542 is the intercept and can be interpreted as the baseline for the Grade 4 cohort in this model. That baseline exceeds the performance-level standard for being categorized as algebra ready. For Grade 4 students who placed Early On Grade Level in all domains, the predicted overall score in Grade 6 was 508, or 34 scale score points lower than the baseline, which exceeds the performance-level standard for on track.

If a Grade 4 student is Mid or Above Grade Level in all domains in winter 2021, their predicted Grade 6 spring 2023 mathematics score is 542, exceeding the performance-level standard for being considered algebra ready. Overall, 5% of students are predicted to be algebra ready by the end of Grade 6. If a Grade 4 student is Mid or Above Grade Level in one domain and Early On Grade Level in three domains, their predicted Grade 6 spring 2023 mathematics score (e.g., 516 or higher) is on track for algebra readiness. Overall, an additional 20% are predicted to be on track. However, if a Grade 4 student is Early On Grade Level in all domains, their predicted Grade 6 spring 2023 mathematics score (e.g., 508) is not on track for algebra readiness. The

model’s R^2 was .62, indicating that the model explained about 62% of the variance in Grade 6 scores.

Table 11. Predicting Grade 6 Overall Mathematics Score Based on Grade 4 Mathematics Domain Placements

Overall Combination	Mid or Above Grade Level	Early On Grade Level	Predicted Overall Mathematics Score in Grade 6	Difference from Baseline
All Domains Mid or Above Grade Level	NO + AL + MS + GEO		542	0
Three Mid or Above Grade Level Domains and One Early On Grade Level Domain	NO + AL + GEO	MS	535	-7
	AL + MS + GEO	NO	534	-8
	NO + AL + MS	GEO	534	-8
	NO + MS + GEO	AL	532	-10
Two Mid or Above Grade Level Domains and Two Early On Grade Level Domains	NO + AL	MS + GEO	526	-16
	MS + GEO	NO + AL	524	-18
One Mid or Above Grade Level Domain and Three Early On Grade Level Domains	AL	NO + MS + GEO	518	-24
	MS	NO + AL + GEO	516	-26
All Domains Early On Grade Level		NO + AL + MS + GEO	508	-34

Note: NO, AL, MS, and GEO refer to Number and Operations, Algebra and Algebraic Thinking, Measurement and Data, and Geometry domains, respectively. The difference from baseline when multiple domains are added may differ slightly from the sum of the differences reported for individual domains due to rounding. More precise point estimates and all combinations of domains can be found in [Appendix B](#).

Table 12 includes results for the Grade 5 cohort. In the Grade 5 cohort, students who placed Mid or Above Grade Level in all four domains in winter of Year 1 were predicted to have an overall mathematics score of 550 by the end of Year 3 (i.e., spring of Grade 7). As such, a score of 550 is the intercept and can be interpreted as the baseline for the Grade 5 cohort in this model. That baseline exceeds the performance-level standard for being categorized as algebra ready. For Grade 5 students who placed Early On Grade Level in all domains, the predicted overall score in Grade 7 was 518, or 32 scale score points lower than the baseline, and does not meet the performance-level standard for either algebra ready or on track.

If a Grade 5 student is Mid or Above Grade Level in at least three domains and Early On Grade Level in one domain in winter 2021, their predicted Grade 7 spring 2023 mathematics score (e.g., 541 or higher) would remain algebra ready. Overall, 12% of students are predicted to be algebra ready by the end of Grade 7. If a Grade 5 student is Mid or Above Grade Level in at least two domains and Early On Grade Level in two domains in winter 2021, their predicted Grade 7 score (e.g., 533 or higher) would exceed 531, passing the performance-level standard for being considered on track for algebra readiness in Grade 5. Overall, an additional 8% of students are predicted to be on track by the end of Grade 7. However, if a Grade 5 student is

Early On Grade Level in three domains and Mid or Above Grade Level in one domain, their predicted Grade 7 mathematics score is not on track for algebra readiness. The model’s R^2 was .61, indicating that the model explained about 61% of the variance in Grade 7 scores.

Table 12. Predicting Grade 7 Overall Mathematics Score Based on Grade 5 Mathematics Domain Placements

Overall Combination	Mid or Above Grade Level	Early On Grade Level	Predicted Overall Mathematics Score in Grade 7	Difference from Baseline
All Domains Mid or Above Grade Level	NO + AL + MS + GEO		550	0
Three Mid or Above Grade Level Domains and One Early On Grade Level Domain	NO + AL + GEO	MS	543	-7
	NO + AL + MS	GEO	543	-7
	NO + MS + GEO	AL	542	-8
	AL + MS + GEO	NO	541	-9
Two Mid or Above Grade Level Domains and Two Early On Grade Level Domains	NO + AL	MS + GEO	536	-14
	MS + GEO	NO + AL	533	-17
One Mid or Above Grade Level Domain and Three Early On Grade Level Domains	NO	AL + MS + GEO	527	-23
	GEO	NO + AL + MS	525	-25
All Domains Early On Grade Level		NO + AL + MS + GEO	518	-32

Note: NO, AL, MS, and GEO refer to Number and Operations, Algebra and Algebraic Thinking, Measurement and Data, and Geometry domains, respectively. The difference from baseline when multiple domains are added may differ slightly from the sum of the differences reported for individual domains due to rounding. More precise point estimates and all combinations of domains can be found in [Appendix B](#).

For students in the Grade 6 cohort, students who placed Mid or Above Grade Level in all four domains in winter of Year 1 were predicted to have an overall mathematics score of 562 by the end of Year 3 (i.e., spring of Grade 8). As such, a score of 562 is the intercept and can be interpreted as the baseline for the Grade 6 cohort in this model. As we also saw in Grades 4 and 5 cohorts, Grade 6 cohort’s baseline score exceeds the placement-level standard for being categorized as algebra ready. As shown in Table 13 for Grade 6 students who placed Early On Grade Level in all domains, the predicted overall score in Grade 8 was 527, or 35 scale score points lower than the baseline, which does not meet or exceed the placement-level standard and, therefore, those students are not predicted to be algebra ready by the end of Grade 8.

If a Grade 6 student is Mid or Above Grade Level in at least two domains and Early On Grade Level in two domains in winter 2021, their predicted Grade 8 spring 2023 mathematics score (e.g., 542 or higher) would be algebra ready. Overall, 17% of students are predicted to be algebra ready by the end of Grade 8. However, if a Grade 6 student is Mid or Above Grade Level in only one domain and Early On Grade Level in three domains in winter 2021, their predicted

Grade 8 mathematics score would not be algebra ready. The model’s R^2 was .58, indicating that the model explained about 58% of the variance in Grade 8 scores.

Table 13. Predicting Grade 8 Overall Mathematics Score Based on Grade 6 Mathematics Domain Placements

Overall Combination	Mid or Above Grade Level	Early On Grade Level	Predicted Overall Mathematics Score in Grade 8	Difference from Baseline
All Domains Mid or Above Grade Level	NO + AL + MS + GEO		562	0
Three Mid or Above Grade Level Domains and One Early On Grade Level Domain	NO + AL + GEO	MS	556	-6
	NO + AL + MS	GEO	554	-8
	AL + MS + GEO	NO	552	-10
	NO + MS + GEO	AL	552	-10
Two Mid or Above Grade Level Domains and Two Early On Grade Level Domains	NO + AL	MS + GEO	547	-15
	MS + GEO	NO + AL	542	-20
One Mid or Above Grade Level Domain and Three Early On Grade Level Domains	NO	AL + MS + GEO	537	-25
	MS	NO + AL + MS	534	-28
All Domains Early On Grade Level		NO + AL + MS + GEO	527	-35

Note: NO, AL, MS, and GEO refer to Number and Operations, Algebra and Algebraic Thinking, Measurement and Data, and Geometry domains, respectively. The difference from baseline when multiple domains are added may differ slightly from the sum of the differences reported for individual domains due to rounding. More precise point estimates and all combinations of domains can be found in [Appendix B](#).

Analysis across Domains

Table 14 compares the domain coefficients based on winter 2021 domain placement stratified by cohort. When considering the point estimates of each domain by cohort, all domains set up students for future algebra success. However, the Algebraic Thinking domain is particularly important for all students. Furthermore, for students in Grades 5 and 6, the Algebraic Thinking and Number and Operations domains are specifically notable.

To elaborate, the further behind any student is in Algebraic Thinking, the faster their predicted score declines. For example, a Grade 2 student’s predicted Grade 4 score drops around 35 scale score points if its winter 2021 domain placement is Two or More Grade Levels Below in Algebraic Thinking compared to dropping 26, 18, and 15 scale score points if its Two or More Grade Levels Below in Number and Operations, Measurement and Data, or Geometry, respectively.

For Grades 5 and 6 students, the further behind a student gets in Algebraic Thinking and Number and Operations, the faster their predicted score declines. To illustrate, a Grade 5 student’s Grade 7 predicted score drops 67 points if its Two or More Grade Levels Below in Algebraic Thinking and Number and Operations, compared to 41 points if its Two or More Grade Levels Below in Measurement and Data and Geometry, respectively.

Table 14. Comparing Domain Coefficients by Winter 2021 Domain Placement Stratified by Cohort

Cohort	Winter 2021 Domain Placement	Domains			
		Number and Operations	Algebra and Algebraic Thinking	Measurement and Data	Geometry
Grades 2 → 4	Early On Grade Level	-5	-7	-6	-5
	One Grade Level Below	-11	-16	-11	-9
	Two or More Grade Levels Below	-26	-35	-18	-15
Grades 3 → 5	Early On Grade Level	-2	-8	-6	-5
	One Grade Level Below	-6	-18	-12	-10
	Two or More Grade Levels Below	-19	-32	-23	-17
Grades 4 → 6	Early On Grade Level	-8	-10	-7	-8
	One Grade Level Below	-12	-18	-13	-13
	Two or More Grade Levels Below	-26	-36	-24	-18
Grades 5 → 7	Early On Grade Level	-9	-8	-8	-7
	One Grade Level Below	-17	-16	-12	-10
	Two or More Grade Levels Below	-34	-33	-23	-18
Grades 6 → 8	Early On Grade Level	-10	-10	-7	-8
	One Grade Level Below	-16	-15	-11	-12
	Two or More Grade Levels Below	-31	-29	-25	-20

Discussion

The results from this analysis provide evidence that domain-level placements in elementary and middle school grades can be used to predict overall mathematics scores two years later. Overall, we found that all domains are critical to future mathematics performance. These findings suggest that taking a holistic approach to mathematics instruction remains the best strategy for preparing students to be algebra ready.

Ultimately, the higher placement level a student started with, the higher placement level they were likely to end with and vice versa. However, in all cohorts, the actual Year 3 placement levels are higher than the predicted placement levels. In other words, in all cohorts, the percentage of students who ended Year 3 on track or algebra ready is higher than the predicted percentage. While we do not know what students’ experiences were between Year 1 and Year 3 assessments, these trends perhaps illustrate the power of intervention and using previous performance to provide students with the appropriate, targeted, and effective instruction.

There are a few limitations to this study. First, a multiple regression model was selected as the method of analysis because it provided easily interpretable results. However, future studies could use structural equation modeling to increase the reliability of the incremental validity estimates (Westfall & Yarkoni, 2016) or multilevel modeling to account for any dependence among scores within schools or school districts (Raudenbush & Bryk, 2002).

Next, this study used Mathematics domain placement levels rather than Mathematics domain scale scores to maintain consistency between grade levels and to provide more interpretable results. However, this reduced the granularity of the mathematics score estimates and meant that those estimates were influenced in part by the range of scale score points included within each placement level by grade. Future analyses could select a different methodology that would utilize scale scores to address a similar research question.

Lastly, we defined algebra ready as students meeting or exceeding 541, which represents the minimum score needed to be considered Mid or Above Grade Level by the end of Grade 8. We could define algebra ready using other performance-level standards. For example, we could have selected the more inclusive performance-level standard, which is equal to one minimum SEM below 541, or a score of 535. Applying this more inclusive performance-level standard to our sample, the percentage of students categorized as algebra ready in cohorts 4 and 5 remain the same or only increase by one percentage point, respectively. However, in cohort 6, using the more inclusive performance-level standard increases the percentage of students algebra ready by five percentage points, or from 17% up to 22%.

Despite these limitations, the findings from this study are more important now than ever. Previous studies have shown that the percentage of students placing on grade level in mathematics decreased substantially after the pandemic and remains stalled (Curriculum Associates, 2023b). Across Grades 1–8, there are fewer students placing on grade level than there were prior to the pandemic, suggesting that far fewer students are on track or algebra ready than ever before. However, there is some good news. The results in this study show that students' starting placement does not have to determine their end placement, especially for the earlier grades. In other words, while students' performance is a critical indicator, it does not predetermine their academic trajectory. Every educator should have access to information on students' domain-level mathematics skills to provide the targeted instruction necessary to best support that student and accelerate their learning. If teachers are empowered with knowledge about where students are and what they need instructionally to be successful, then perhaps they can accelerate students' mathematics trajectories toward algebra readiness.

Full Report References

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Appendix A: Percentage Table of Domain Placement Combinations

Tables show a selection of domain combinations and the proportion of students in the cohort represented by these combinations.

Table A1. Grade 2 Cohort—Percentage Table of Domain Placement Combinations

Winter 2021 Domain Placement Level					
Overall Combination	Mid or Above Grade Level	Early On Grade Level	One Grade Level Below	Number of Students	Percentage (%)
4 Mid-Above	NO + AL + MS + GEO			8,692	4.85
3 Mid-Above and 1 Early-On	AL + MS + GEO	NO		3,794	2.12
	NO + MS + GEO	AL		3,320	1.85
	NO + AL + GEO	MS		2,015	1.12
	NO + AL + MS	GEO		1,436	.80
2 Mid-Above and 2 Early-On	NO + AL	MS + GEO		511	.28
	MS + GEO	NO + AL		2,927	1.63
	NO + MS	AL + GEO		732	.41
	NO + GEO	AL + MS		1,497	.83
	AL + MS	NO + GEO		787	.44
	AL + GEO	NO + MS		1,465	.82
1 Mid-Above and 3 Early-On	NO	AL + MS + GEO		422	.24
	AL	NO + MS + GEO		418	.23
	MS	NO + AL + GEO		1,399	.78
	GEO	NO + AL + MS		1,578	.88
4 Early-On		NO + AL + MS + GEO		838	.47
3 Early-On and 1 One-Below		AL + MS + GEO	NO	665	.37
		NO + MS + GEO	AL	663	.37
		NO + AL + GEO	MS	1,443	.80
		NO + AL + MS	GEO	800	.45
2-Early-On and 2 One-Below		NO + AL	MS + GEO	2,089	1.16
		MS + GEO	NO + AL	738	.41
		NO + MS	AL + GEO	1,321	.74
		NO + GEO	AL + MS	1,826	1.02
		AL + MS	NO + GEO	1,314	.73
		AL + GEO	NO + MS	1,669	.93
1 Early-On and 3 One-Below		NO	AL + MS + GEO	5,101	2.84
		AL	NO + MS + GEO	4,866	2.71
		MS	NO + AL + GEO	3,128	1.74
		GEO	NO + AL + MS	2,964	1.65
4 One-Below			NO + AL + MS + GEO	23,373	13.03

Note: NO, AL, MS, and GEO refer to Number and Operations, Algebra and Algebraic Thinking, Measurement and Data, and Geometry domains, respectively.

Table A2. Grade 3 Cohort—Percentage Table of Domain Placement Combinations

Winter 2021 Domain Placement Level					
Overall Combination	Mid or Above Grade Level	Early On Grade Level	One Grade Level Below	Number of Students	Percentage (%)
4 Mid-Above	NO + AL + MS + GEO			19,795	4.98
3 Mid-Above and 1 Early-On	AL + MS + GEO	NO		6,081	1.53
	NO + MS + GEO	AL		2,080	.52
	NO + AL + GEO	MS		1,969	.50
	NO + AL + MS	GEO		6,900	1.74
2 Mid-Above and 2 Early-On	NO + AL	MS + GEO		1,625	.41
	MS + GEO	NO + AL		2,020	.51
	NO + MS	AL + GEO		1,416	.36
	NO + GEO	AL + MS		748	.19
	AL + MS	NO + GEO		4,045	1.02
	AL + GEO	NO + MS		1,752	.44
1 Mid-Above and 3 Early-On	NO	AL + MS + GEO		669	.17
	AL	NO + MS + GEO		1,501	.38
	MS	NO + AL + GEO		1,352	.34
	GEO	NO + AL + MS		1,147	.29
4 Early-On		NO + AL + MS + GEO		904	.23
3 Early-On and 1 One-Below		AL + MS + GEO	NO	1,200	.30
		NO + MS + GEO	AL	567	.14
		NO + AL + GEO	MS	1,197	.30
		NO + AL + MS	GEO	2,389	.60
2-Early-On and 2 One-Below		NO + AL	MS + GEO	5,375	1.35
		MS + GEO	NO + AL	1,153	.29
		NO + MS	AL + GEO	2,947	.74
		NO + GEO	AL + MS	1,323	.33
		AL + MS	NO + GEO	5,947	1.50
		AL + GEO	NO + MS	2,281	.57
1 Early-On and 3 One-Below		NO	AL + MS + GEO	8,918	2.24
		AL	NO + MS + GEO	14,524	3.65
		MS	NO + AL + GEO	8,341	2.10
		GEO	NO + AL + MS	3,779	.95
4 One-Below			NO + AL + MS + GEO	41,459	10.43

Note: NO, AL, MS, and GEO refer to Number and Operations, Algebra and Algebraic Thinking, Measurement and Data, and Geometry domains, respectively.

Table A3. Grade 4 Cohort—Percentage Table of Domain Placement Combinations

Winter 2021 Domain Placement Level					
Overall Combination	Mid or Above Grade Level	Early On Grade Level	One Grade Level Below	Number of Students	Percentage (%)
4 Mid-Above	NO + AL + MS + GEO			6,380	5.03
3 Mid-Above and 1 Early-On	AL + MS + GEO	NO		912	.72
	NO + MS + GEO	AL		404	.32
	NO + AL + GEO	MS		766	.60
	NO + AL + MS	GEO		3,993	3.15
2 Mid-Above and 2 Early-On	NO + AL	MS + GEO		1,275	1.01
	MS + GEO	NO + AL		209	.16
	NO + MS	AL + GEO		777	.61
	NO + GEO	AL + MS		184	.15
	AL + MS	NO + GEO		1,769	1.40
	AL + GEO	NO + MS		353	.28
1 Mid-Above and 3 Early-On	NO	AL + MS + GEO		571	.45
	AL	NO + MS + GEO		1,212	.96
	MS	NO + AL + GEO		884	.70
	GEO	NO + AL + MS		178	.14
4 Early-On		NO + AL + MS + GEO		778	.61
3 Early-On and 1 One-Below		AL + MS + GEO	NO	512	.40
		NO + MS + GEO	AL	482	.38
		NO + AL + GEO	MS	862	.68
		NO + AL + MS	GEO	1,871	1.48
2-Early-On and 2 One-Below		NO + AL	MS + GEO	2,374	1.87
		MS + GEO	NO + AL	357	.28
		NO + MS	AL + GEO	1,268	1.00
		NO + GEO	AL + MS	725	.57
		AL + MS	NO + GEO	1,337	1.05
		AL + GEO	NO + MS	661	.52
1 Early-On and 3 One-Below		NO	AL + MS + GEO	3,119	2.46
		AL	NO + MS + GEO	2,698	2.13
		MS	NO + AL + GEO	1,354	1.07
		GEO	NO + AL + MS	886	.70
4 One-Below			NO + AL + MS + GEO	5,660	4.46

Note: NO, AL, MS, and GEO refer to Number and Operations, Algebra and Algebraic Thinking, Measurement and Data, and Geometry domains, respectively.

Table A4. Grade 5 Cohort—Percentage Table of Domain Placement Combinations

Winter 2021 Domain Placement Level					
Overall Combination	Mid or Above Grade Level	Early On Grade Level	One Grade Level Below	Number of Students	Percentage (%)
4 Mid-Above	NO + AL + MS + GEO			5,316	5.45
3 Mid-Above and 1 Early-On	AL + MS + GEO	NO		712	.73
	NO + MS + GEO	AL		2,191	2.25
	NO + AL + GEO	MS		440	.45
	NO + AL + MS	GEO		2,642	2.71
2 Mid-Above and 2 Early-On	NO + AL	MS + GEO		458	.47
	MS + GEO	NO + AL		885	.91
	NO + MS	AL + GEO		1,861	1.91
	NO + GEO	AL + MS		423	.43
	AL + MS	NO + GEO		663	.68
	AL + GEO	NO + MS		165	.17
1 Mid-Above and 3 Early-On	NO	AL + MS + GEO		538	.55
	AL	NO + MS + GEO		218	.22
	MS	NO + AL + GEO		1,201	1.23
	GEO	NO + AL + MS		277	.28
4 Early-On		NO + AL + MS + GEO		590	.60
3 Early-On and 1 One-Below		AL + MS + GEO	NO	546	.56
		NO + MS + GEO	AL	599	.61
		NO + AL + GEO	MS	500	.51
		NO + AL + MS	GEO	1,150	1.18
2-Early-On and 2 One-Below		NO + AL	MS + GEO	1,148	1.18
		MS + GEO	NO + AL	573	.59
		NO + MS	AL + GEO	1,175	1.20
		NO + GEO	AL + MS	752	.77
		AL + MS	NO + GEO	930	.95
		AL + GEO	NO + MS	614	.63
1 Early-On and 3 One-Below		NO	AL + MS + GEO	2,207	2.26
		AL	NO + MS + GEO	1,927	1.98
		MS	NO + AL + GEO	2,275	2.33
		GEO	NO + AL + MS	1,297	1.33
4 One-Below			NO + AL + MS + GEO	6,393	6.55

Note: NO, AL, MS, and GEO refer to Number and Operations, Algebra and Algebraic Thinking, Measurement and Data, and Geometry domains, respectively.

Table A5. Grade 6 Cohort—Percentage Table of Domain Placement Combinations

Winter 2021 Domain Placement Level					
Overall Combination	Mid or Above Grade Level	Early On Grade Level	One Grade Level Below	Number of Students	Percentage (%)
4 Mid-Above	NO + AL + MS + GEO			3,151	4.52
3 Mid-Above and 1 Early-On	AL + MS + GEO	NO		401	.57
	NO + MS + GEO	AL		930	1.33
	NO + AL + GEO	MS		284	.41
	NO + AL + MS	GEO		1,715	2.46
2 Mid-Above and 2 Early-On	NO + AL	MS + GEO		358	.51
	MS + GEO	NO + AL		578	.83
	NO + MS	AL + GEO		1,394	2.00
	NO + GEO	AL + MS		175	.25
	AL + MS	NO + GEO		654	.94
	AL + GEO	NO + MS		88	.13
1 Mid-Above and 3 Early-On	NO	AL + MS + GEO		427	.61
	AL	NO + MS + GEO		176	.25
	MS	NO + AL + GEO		855	1.23
	GEO	NO + AL + MS		176	.25
4 Early-On		NO + AL + MS + GEO		575	.82
3 Early-On and 1 One-Below		AL + MS + GEO	NO	448	.64
		NO + MS + GEO	AL	567	.81
		NO + AL + GEO	MS	382	.55
		NO + AL + MS	GEO	579	.83
2-Early-On and 2 One-Below		NO + AL	MS + GEO	518	.74
		MS + GEO	NO + AL	536	.77
		NO + MS	AL + GEO	771	1.10
		NO + GEO	AL + MS	406	.58
		AL + MS	NO + GEO	627	.90
		AL + GEO	NO + MS	416	.60
1 Early-On and 3 One-Below		NO	AL + MS + GEO	1,053	1.51
		AL	NO + MS + GEO	964	1.38
		MS	NO + AL + GEO	1,222	1.75
		GEO	NO + AL + MS	922	1.32
4 One-Below			NO + AL + MS + GEO	3,949	5.66

Note: NO, AL, MS, and GEO refer to Number and Operations, Algebra and Algebraic Thinking, Measurement and Data, and Geometry domains, respectively.

Appendix B: Results of the Multiple Regression Model

The full results of the multiple regression model for including intercepts and domain by placement level point estimates for cohorts Grades 2–6 are presented in Tables B1–B5. The intercept can be interpreted as the estimated Year 3 mathematics score for a student who scored Mid or Above Grade Level in all domains assessed in Year 1. The estimated Year 3 mathematics score for a student with one or more domain-specific placements in Year 1 other than Mid or Above Grade Level can be calculated by adding the point estimate associated with that domain-specific placement to the intercept. Note that the point estimate reported in the B column sometimes differs from the Difference from Baseline column in Tables 8–12 due to rounding.

Table B1: Grade 2 Cohort—Results of the Multiple Regression Model

		<i>B</i>	<i>SE</i>	<i>t</i>	<i>p</i>
Grade 4 Intercept (Predicted Score When All Grade 2 Domain Placements Are Mid or Above Grade Level)		509.68	.15	3367.99	< .01
Number and Operations	Early On Grade Level	-4.67	.17	-27.72	< .01
	One Grade Level Below	-11.20	.18	-63.89	< .01
	Two or More Grade Levels Below	-25.80	.25	-102.20	< .01
Algebra and Algebraic Thinking	Early On Grade Level	-7.37	.17	-44.40	< .01
	One Grade Level Below	-15.63	.17	-89.82	< .01
	Two or More Grade Levels Below	-34.61	.27	-126.13	< .01
Measurement and Data	Early On Grade Level	-5.78	.18	-32.80	< .01
	One Grade Level Below	-10.76	.16	-68.74	< .01
	Two or More Grade Levels Below	-17.63	.21	-82.33	< .01
Geometry	Early On Grade Level	-4.59	.17	-26.33	< .01
	One Grade Level Below	-8.68	.16	-55.52	< .01
	Two or more Grade Levels Below	-15.01	.20	-73.86	< .01
N	179,341				
<i>R</i> ²	.55				

Table B2: Grade 3 Cohort—Results of the Multiple Regression Model

		<i>B</i>	<i>SE</i>	<i>t</i>	<i>p</i>
Grade 5 Intercept (Predicted Score When All Grade 3 Domain Placements Are Mid or Above Grade Level)		520.56	.12	4422.77	< .01
Number and Operations	Early On Grade Level	-1.79	.13	-13.40	< .01
	One Grade Level Below	-5.67	.12	-45.93	< .01
	Two or More Grade Levels Below	-18.91	.18	-102.97	< .01
Algebra and Algebraic Thinking	Early On Grade Level	-8.06	.12	-65.98	< .01
	One Grade Level Below	-17.75	.12	-148.21	< .01
	Two or More Grade Levels Below	-32.33	.18	-182.48	< .01
Measurement and Data	Early On Grade Level	-5.61	.13	-41.99	< .01
	One Grade Level Below	-12.35	.12	-106.52	< .01
	Two or More Grade Levels Below	-22.78	.16	-146.90	< .01
Geometry	Early On Grade Level	-4.96	.15	-33.54	< .01
	One Grade Level Below	-10.13	.13	-79.77	< .01
	Two or More Grade Levels Below	-17.26	.17	-100.91	< .01
N	397,591				
<i>R</i> ²	.54				

Table B3. Grade 4 Cohort—Results of the Multiple Regression Model

		<i>B</i>	<i>SE</i>	<i>t</i>	<i>p</i>
Grade 6 Intercept (Predicted Score When All Grade 4 Domain Placements Are Mid or Above Grade Level)		541.95	.22	2441.99	< .01
Number and Operations	Early On Grade Level	-8.13	.20	-40.26	< .01
	One Grade Level Below	-12.31	.22	-55.56	< .01
	Two or More Grade Levels Below	-25.62	.29	-88.15	< .01
Algebra and Algebraic Thinking	Early On Grade Level	-9.72	.20	-49.43	< .01
	One Grade Level Below	-18.37	.21	-87.66	< .01
	Two or More Grade Levels Below	-36.48	.29	-127.50	< .01
Measurement and Data	Early On Grade Level	-7.21	.21	-34.36	< .01
	One Grade Level Below	-12.51	.20	-61.91	< .01
	Two or More Grade Levels Below	-23.48	.27	-88.70	< .01
Geometry	Early On Grade Level	-8.42	.27	-30.86	< .01
	One Grade Level Below	-12.80	.27	-48.05	< .01
	Two or More Grade Levels Below	-18.19	.30	-60.31	< .01
N	126,806				
<i>R</i> ²	.62				

Table B4. Grade 5 Cohort—Results of the Multiple Regression Model

		<i>B</i>	<i>SE</i>	<i>t</i>	<i>p</i>
Grade 7 Intercept (Predicted Score When All Grade 5 Domain Placements Are Mid or Above Grade Level)		550.30	.24	2260.73	< .01
Number and Operations	Early On Grade Level	-8.89	.26	-34.29	< .01
	One Grade Level Below	-17.05	.27	-62.82	< .01
	Two or More Grade Levels Below	-34.37	.39	-88.00	< .01
Algebra and Algebraic Thinking	Early On Grade Level	-8.49	.27	-31.54	< .01
	One Grade Level Below	-16.13	.29	-56.34	< .01
	Two or More Grade Levels Below	-32.58	.38	-86.42	< .01
Measurement and Data	Early On Grade Level	-7.65	.26	-28.97	< .01
	One Grade Level Below	-12.08	.26	-46.28	< .01
	Two or More Grade Levels Below	-23.42	.33	-70.47	< .01
Geometry	Early On Grade Level	-6.84	.29	-23.71	< .01
	One Grade Level Below	-10.49	.29	-35.99	< .01
	Two or More Grade Levels Below	-17.61	.34	-51.30	< .01
N	97,562				
<i>R</i> ²	.61				

Table B5. Grade 6 Cohort—Results of the Multiple Regression Model

		<i>B</i>	<i>SE</i>	<i>t</i>	<i>p</i>
Grade 8 Intercept (Predicted Score When All Grade 6 Domain Placements Are Mid or Above Grade Level)		562.44	.35	1628.56	< .01
Number and Operations	Early On Grade Level	-10.05	.36	-28.29	< .01
	One Grade Level Below	-15.88	.37	-43.03	< .01
	Two or More Grade Levels Below	-31.43	.48	-65.22	< .01
Algebra and Algebraic Thinking	Early On Grade Level	-10.02	.37	-27.03	< .01
	One Grade Level Below	-15.47	.39	-39.78	< .01
	Two or More Grade Levels Below	-28.96	.48	-59.89	< .01
Measurement and Data	Early On Grade Level	-6.88	.35	-19.71	< .01
	One Grade Level Below	-11.14	.35	-31.69	< .01
	Two or More Grade Levels Below	-24.70	.44	-56.45	< .01
Geometry	Early On Grade Level	-8.22	.39	-21.29	< .01
	One Grade Level Below	-11.85	.40	-29.97	< .01
	Two or More Grade Levels Below	-19.97	.47	-42.21	< .01
N	69,787				
<i>R</i> ²	.58				