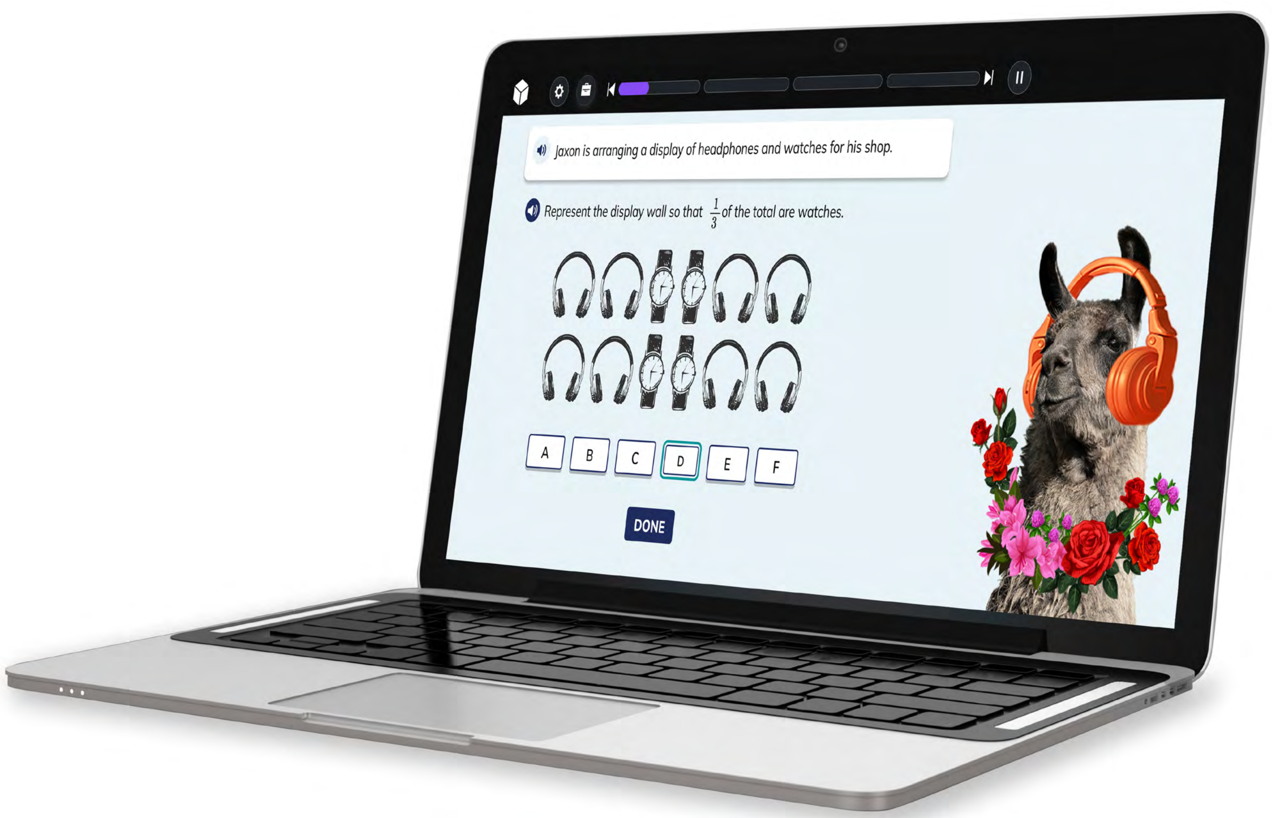
Essential Lessons in Core Numeracy Research Base

Mathematics



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INTRODUCTION

Essential Lessons are curricula specifically designed to focus on core numeracy skills and developing a strong number sense to make meaning of mathematics for older striving learners. Older striving learners are defined as students in Grades 6–8 who place two or more grade levels behind in mathematics. This program gives learners in this category the ability to master foundational mathematical concepts to prepare them for engagement in grade-level content.

The Essential Lessons curriculum includes:

- New lessons to drive student focus and engagement
- Accessibility enhancements supporting all students
- Increased adaptivity in mathematics, which ensures students receive the instruction they need when they need it

Because older striving learners need differentiated support, Essential Lessons offer streamlined lesson pathways, a reimagined experience, and deeper insights, which together offer adaptivity and autonomy. This approach elevates students to grade level and increases motivation. The following pages outline the research base upon which Essential Lessons was built and how it delivers explicit and systematic foundational mathematical concepts to older striving learners.

PROGRAM OVERVIEW

Essential Lessons, the next evolution of *i-Ready Pro* for middle school, offer a reimagined student experience with streamlined lesson pathways and deep insights for educators into skills progress and development. Essential Lessons give older striving learners access to efficient, focused, and evidence-based intervention lessons that support them in building fundamental numeracy skills and conceptual understanding.

Lessons are designed specifically to meet the needs of middle school students, with a more mature design and sleeker look and feel. Lessons motivate and engage older striving learners as they work through the lessons, and students have more freedom and control over what and how they are learning. Students who demonstrate proficiency in concepts progress to more advanced content quicker, while students who need additional support get the explicit instruction and practice they need.

Essential Lessons include roughly 30 lessons with a focus on teaching students to work flexibly with whole numbers, fractions, and decimals, and to perform operations with these numbers.

Each lesson consists of two skills. For each skill, there is an Exploration, Application, and Retry section:

- **Exploration:** Tasks are used to allow students the opportunity to dive into the investigation question as they explore the mathematical concept. In the end, learners "unlock" the key idea by revealing a key takeaway.

- **Application:** Learners apply the mathematical concept in a contextual situation. The tasks in this section are scored to measure the learner's level of skill mastery. The learner is given an initial task, referred to as the A task. If the learner successfully completes the task, they move to another A task. If they are not successful, they are given a B task that is equal in rigor and identical in structure to the A task but uses different numbers and possibly a different context.

- **Retry:** At the end of the lesson, the learner can go back to any skills that have not been demonstrated in the Application section. The learner is given an option to retry the skill(s) without having to go back through the entire lesson.

The lessons together are designed to take roughly a year or less for most students, depending on the amount of additional scaffolding and practice a student needs.

THE EVIDENCE BASE

Importance of Addressing Older Striving Learners

By the time students arrive in middle school, there are fundamental mathematics concepts and skills that they have demonstrated in order to be able to access a more advanced middle school curriculum. However, some students may arrive in middle school without the deep conceptual understanding of numeracy and other skill areas that they need to be able to intellectually advance to more complex concepts.

In *Build a Bridge*: *Provide Access to Grade-Level Content for All Students*, Coleman (2020) describes two options for supporting struggling learners in middle school: remediation and teaching grade-level content without mastering previous years' content. As she explains, neither of these options meets the needs of the learner in a meaningful way. A third option, as introduced in the article's name, is to *build a bridge*. Essential Lessons are based on the concept that there is a third way to address older striving learners by providing the concepts without the program feeling like remediation.

These lessons allow schools and districts to support their older striving learners with content and lessons that have age-appropriate language, examples, and images while still filling in the most important conceptual gaps in their learning.

The Institute of Education Sciences (IES) What Works Clearinghouse paper *Assisting Students Struggling with Mathematics*: *Response to Intervention* (RtI) *for Elementary and Middle Schools* recommends that instructional materials for students receiving interventions focus intensely on in-depth treatment on numeracy and rational numbers in Grades 4 through 8 (IES Practice Guide, 2009). Essential Lessons build skills around numeracy and use them as a bridge to other mathematical domains.

Research on Growth Mindset

A substantial body of research demonstrates that instruction that supports a growth mindset by encouraging persistence of effort is more effective than reinforcing the idea that intelligence and abilities are fixed (Dweck, 2015).

Strong Research Support for a Growth Mindset

Older striving learners are likely to have felt discouraged in their elementary school experience, increasing the need to build a growth mindset in middle school. According to Dweck's (2015) research, students who believe their intelligence is a fixed construct are less motivated to apply themselves in school. They become more focused on appearing to be smart, and their beliefs make them see challenges and the need to exert effort as more of a threat than a way to improve their learning. By praising innate intelligence, teachers and parents reinforce students' views that their abilities are fixed rather than malleable and discourage students from striving to improve (Dweck, 2015).

Researchers have carried out multiple studies examining how students' theories about their own intelligence impacts their learning (Blackwell et al., 2007). Studies by Dweck found that the students who were encouraged to succeed through increased effort and persistence after repeated failure came to see their failure as being due to insufficient effort rather than innate ability. By not viewing their intelligence as static, these students came to understand that if they persevered, they could succeed at tasks at which they had once failed (Dweck, 2015). Essential Lessons support a growth mindset through their emphasis on concepts rather than focusing on students' lesson scores. Specific elements of Essential Lessons that reinforce growth mindset include:

- They are adaptable and show students that part of learning includes making mistakes. For example, students who need support receive multiple additional opportunities to show they can perform a skill.

- There are different types of interactive feedback throughout the lessons to support students as they work through the tasks. One example of this support is walkthroughs, in which students who answer incorrectly are led through a task in a step-by-step format.

- Students who need support with a skill on their first try receive feedback, followed by a second, similar task, which gives them a chance to demonstrate growth on the skill.

Two studies focused on 373 junior high school students over a two-year period to investigate how students' mindsets might impact their learning in mathematics. At the start of the first study, students' mindsets were assessed, and the researchers found that students with a growth mindset tended to experience an upward trajectory in grades. Students who believed their intelligence was fixed tended toward a flat trajectory. The second study designed an intervention to teach students how to develop a growth mindset. The results indicated that teaching students to have a growth mindset (referred to as an "incremental theory") resulted in improved motivation and a reversal in the downward trajectory of their grades. Students in the control group were less motivated and continued on a downward trajectory in grades (Blackwell et al., 2007).

Blackwell et al. (2007) also concluded that children begin developing fixed or growth mindsets in elementary school, but often, problems do not arise until academic work becomes more challenging during early adolescence. Because elementary school is more failure-proof, students with a fixed mindset are less apt to experience negative consequences until they reach middle school.

Effect of Mathematics Anxiety on Growth Mindset

Essential Lessons give the middle school student the opportunity to experience success, decreasing their mathematics anxiety and increasing their numerical knowledge. Boaler (2017; 2019) explains that many children in the US, and beyond, grow up thinking that either you can do math or you can't. When they need support, they cannot do math because of this need for support. Expectations are very powerful in developing a growth mindset, and this need for support creates widespread mathematics anxiety. One study found that 48 percent of all young adults in a work-apprentice program had mathematics anxiety, whereas other studies have found that approximately 50 percent of students taking introductory mathematics courses in college suffer from mathematics anxiety. The way to address mathematics anxiety is to teach mathematics as a multidimensional subject and offer different entry points for students. Anxiety can be alleviated by experiencing success in mathematics (Luttenberger et al., 2018).

Importance of Motivation

The importance of connecting motivation to learning is evident throughout Essential Lessons. Self-determined motivation (a consequence of values or pure interest) leads to better long-term outcomes than controlled motivation (a consequence of reward/punishment or perceptions of self-worth) (Davis et al., 2006). Contexts and activities that support students' feelings of having agency and competence, and seeing the relevance of what they are doing have been shown to result in high-quality, self-directed, intrinsic motivation (Niemiec & Ryan, 2009).

Research has shown that boredom is negatively correlated with learning (Craig et al., 2004). Pedagogically motivated learning modifications may lead to increased learning in the short run; however, if learners find the modifications boring, they may not learn as much in the long run (Jackson & McNamara, 2011; 2013). Therefore, sustained motivation during lessons is a key focus of Essential Lessons. Essential Lessons build motivation by giving students autonomy, increasing their feelings of competence, and creating a learning experience that is relevant and that students value.

Provides Opportunities for Autonomy

When students feel they have autonomy, they have positive feelings about the task they are being asked to complete and are therefore more likely to value the task (Grolnick et al., 1991; Grolnick & Ryan, 1989). Additionally, students who experience autonomy are more likely to show strong behavioral and cognitive engagement (Grolnick et al., 1997; Deci et al., 1996; Deci et al., 1991; Connell, 1990). Actions that support autonomy include providing choice and encouraging self-initiation/self-direction (Reeve et al., 2003; Assor et al., 2002).

Essential Lessons encourage students' autonomy through:

- **Choice of scaffolding.** After repeated need for support, students can choose to receive scaffolded support (depending on the task, this might be an interactive walkthrough (e.g., Step It Out) or explanation of the solution (e.g., Show Me) or keep solving on their own.

- **Choice to retry.** When a student completes a lesson, they can see which skills they successfully acquired and where they could benefit from additional instruction and practice. Students can choose whether they want to retry the skills they need support with. If they retry, they receive a shorter Retry lesson designed to better scaffold striving students to succeed at the skills they missed.

Increases Feelings of Competence

Working at the right level of challenge and avoiding excessively easy or hard challenges (i.e., working in the zone of proximal development) enhances competence and increases a student's potential for new learning (Cook & Artino, 2016; Fabes & Martin, 2001). Texts and tasks that are scaffolded by the teacher and are "just challenging enough" help students both cognitively and affectively (Robertson et al., 2014). Encouragement and positive feedback increase feelings of competence (Reeve & Jang, 2006; Ryan, 1982). Tracking students' progress increases interactions between teachers and students and provides better guidance for students to enhance their learning (Marzano, 2010).

Essential Lessons increase feelings of competence through:

- **Redemption opportunities.** After needing support on a practice problem, students always get a second chance at a similar problem, providing them with an opportunity to demonstrate and feel competence.

- **Educator feedback on skills performance.** As students progress through the scope and sequence, teachers are alerted when students show they consistently need support so they can intervene when the student has moved on to content that may be too hard for them.

- **Positive feedback.** Throughout the lessons, students are given incremental positive reinforcement each time they get an item correct.

Creates Relevant Learning Experiences

There are a few factors that drive a student's decision to invest in a learning activity. They include the importance students attach to doing well on a task, the enjoyment they experience as a result of participation, the usefulness of participating toward future goals, and the extent to which participating in one activity may come at the expense of another (Juvonen et al., 2012; Carter, 2006).

In designing materials that capture and excite a learner, research has shown that designers have only 25–35 seconds to capture and engage a user's attention (Nielsen & Loranger, 2006). Research also shows that older students can quickly become bored, distracted, and frustrated. Regarding digital content, they can also be easily overwhelmed with website content, and cluttered screens can distract them and deter learning. Because older striving students are the audience for Essential Lessons, it is especially important to provide age-appropriate content with neutral graphics rather than childish ones (Joyce & Harley, 2005).

Essential Lessons create a relevant learning experience that is:

- **Designed for older students.** Essential Lessons are designed specifically for Grades 6-8 students. Everything from the color palettes to the interactions were tested with students to find an experience that would appeal to them and feel age appropriate. Animations are designed to be fun and age appropriate. Instruction is provided efficiently and in short bursts to hold students' attention, and only when students demonstrate a need.

- **Uncluttered digital learning.** The screens that students interact with are clean and uncluttered, allowing the learner to focus solely on each instructional task that they are presented with across the lesson.

Committing to Learner Variability and Equity

Curriculum Associates has a strong commitment to centering equity in all that we do. We believe that all students deserve access to high-quality, equitable educational resources and classroom instruction. We recognize that every classroom represents a rich diversity of culture, linguistic backgrounds, economic status, living circumstances, and ability, and that each student represents unique intersections between these various aspects of identity. Essential Lessons strive to ensure that our materials and texts reflect and honor our students as individuals and readers, and value all that they bring to the classroom.

Recognizing the diversity of today's classrooms, Essential Lessons employ various strategies to support and include all students. Informed by leading science research and cognitive neuroscience, learner variability suggests there is no average learner and all students bring their own unique assets, backgrounds, and variables to their learning (Dockterman, 2018; Rose et al., 2013).

Universal Design for Learning

The Universal Design for Learning (UDL) is a scientifically valid framework for guiding educational practices that anticipates learner variability, removes barriers in instruction, and incorporates goal setting and reflection to create "expert learners" who are purposeful and motivated, strategic and goal-directed, and resourceful and knowledgeable (CAST, 2018). The UDL is built on decades of research in neuroscience and is grounded on the foundation of three principles that remind educators to provide students with options for personalizing their education: (1) multiple means of engagement; (2) multiple means of representation; and (3) multiple means of action and expression (Chardin & Novak, 2020).

Essential Lessons offer alternatives for auditory information by providing closed captioning and audiovisual syncing for all instruction and feedback. Alternatives for visual information include spoken descriptions and alt text for all images. Instruction at the start of each lesson activates students' prior knowledge, highlights helpful strategies and skills to use, and includes explicit modeling of prerequisite skills. Essential Lessons integrate mouse and keyboard navigation to ensure all students can navigate and interact with lesson screens and provide a seamless interface with many common assistive technologies, such as screen readers.

Standards-Based Mathematics Instruction

Two sets of curriculum standards have been largely responsible for driving reform in mathematics instruction across the United States: the National Council of Teachers of Mathematics (NCTM)'s *Principles and Standards for School Mathematics* (NCTM, 2000) and the Common Core State Standards (CCSS) for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2019). Most states' curriculum standards have been informed by one or both of these works, including states that have not adopted the CCSS and states that have revised their standards after having adopted the CCSS.

In 2014, the NCTM published *Principles to Actions*: *Ensuring Mathematical Success for All*, which recommends actions for mathematics educators, school and district administrators, other education leaders, and policymakers reflecting the CCSS and more than a decade of experience and new research evidence about excellent mathematics programs (NCTM, 2014).

Together, these and related documents provide research-informed guidance on the "what" and "how" of high-quality mathematics instruction.

The work of the NCTM and the CCSS Initiative's Mathematics team can be seen as a back-and-forth, iterative process. The NCTM's *Principles and Standards for School Mathematics* (2000) was a foundational source for the authors of the CCSS for Mathematics (2010). More recently, the NCTM's *Principles to Actions* (2014) provided guidance to the education community on research-based best teaching practices for implementation of the CCSS.

Analysis across the work of the NCTM and the CCSS Initiative suggests the following are some of the common standards-based recommendations:

- Instruction should promote sense making and understanding of mathematical concepts and procedures in the context of meaningful problem solving.
- Sense making requires encouragement of and guidance on reasoning about relationships among mathematics concepts and procedures as well as on recognizing patterns and structure within problem situations, so students come to see mathematics as a system of interrelated ideas.
- Instruction should support student interpretation and development of simplified models of real-life problems, represented in multiple ways—graphically, symbolically, and verbally.
- Instructional activities should encourage and provide support for perseverance and productive struggle when attempting to solve challenging mathematics problems.

Research on Meaning Making in Mathematics

Research and expert opinion support the conclusion that the act of meaning making is essential for successful mathematics learning to take place and that helping students see structural relationships is key to meaning making. There is consensus among researchers that students can better construct meaning in the context of mathematical problem solving. Successful mathematics instruction has shifted toward supporting more meaningful learning by building on students' entry knowledge, providing opportunities for intervention and practice, examining multiple problem-solving methods, and calling on students to provide explanations of methods and why they work. Skills development remains important, but it needs to be incorporated into students' construction of knowledge.

Learning as Meaning Making

Meaning making is the ability for a learner to understand their surroundings and the world around them as a way to motivate and ground learning. As we grow, our mind develops to construct meaning, which ultimately leads to a learner being able to successfully grasp content (Sfard, 2003). Structure is essential for developing understanding and making meaning.

Prominent theorists have weighed in on this important relationship between seeing structural relationships and developing understanding: The idea of understanding [is] almost tantamount to seeing relationships (Sfard, 2003). Sfard (2003) cites Skemp's (1976) distinction between know[ing] rules without reasons and knowing both what to do and why. In mathematics, knowing why requires understanding the structural relationships among the facts, concepts, and procedures. The authors of the NCTM Standards link the standards to mathematical structures and support using problem scenarios as relevant contexts for extracting mathematical structure.

In Essential Lessons, students make meaning of the underlying concepts they are learning in the Exploration section. Questions during exploration are often designed to motivate conceptual discussion. These questions encourage students to think critically about the mathematical concept being addressed rather than applying a formula or algorithm they may not fully understand. The end result is a deeper understanding and an opportunity for students to uncover misconceptions they may have around the concept. The investigations in the Exploration section create a sense of meaning and connection between the learner and the content.

Importance of Developing Number Sense

Research for decades has recognized the importance of all learners developing strong number sense. Number sense is the ability to understand operation and numbers and use that understanding to develop strategies to solve mathematics problems, and these skills are the backbone of understanding and progressing in mathematics. This understanding of numbers allows for a student to communicate, process, and interpret mathematical information (Sood & Mackey, 2014).

Just as our understanding of phonemic awareness has revolutionized the teaching of beginning reading, the influence of number sense on early mathematics development and more complex mathematical thinking carries implications for instruction (Gersten & Chard, 1999).

Research shows that sense making of mathematics concepts leads to engagement and understanding: With well-developed number sense knowledge, students can use flexible ways to make mathematical judgments and develop useful strategies for solving complex problems. Students who lack a strong number sense have trouble developing the foundation needed for even simple arithmetic, let alone more complex mathematics (Battista, 2017).

Twenty years ago, in the study *Developing Number Sense: An Intervention Study in Grade 7*, Zvia Markovits and Judith Sowder (1994) showed that when Grade 7 students were taught number magnitude, mental computation, and computational estimation, they were able to use these strategies in later lessons to support them in understanding the content. The study used written measures and interviews before and after instruction. Several months later, the study revealed that after instruction, students were more likely to elect to use strategies that reflected number sense, and that this was a long-term change.

In a recent paper on number sense focused on algebra/pre-algebra students, the authors note that a large percentage of study participants (i.e., middle school students) demonstrate poor understanding of rational numbers; inability to make good quantitative judgments; lack of reasoning skills; reliance solely on rule-based procedures to arrive at an answer; deficiency in estimation skills; and inability to perform mental calculations (Parveen, 2014).

In *A New Vision for Developing Addition and Subtraction Computation Strategies* by Randall I. Charles (2020), he defines four key outcomes of improved numbers sense in students:

- Alternative computation strategies **promote flexibility**. The operation and the nature of the numbers involved inform which **calculation strategy** is most efficient and accurate.
- Alternative computation strategies **emphasize mental math** with recordings used to support thinking.
- Alternative computation strategies invite **multiple methods** to be used for the same calculation.
- Alternative computation strategies focus on **number reasoning** that is based on place value relationships, meanings of operations, relationships among operations, and properties.

Essential Lessons' goal is to engage older striving learners, assess their gaps in understanding, and offer targeted lessons to build numeracy skills. Without a strong grasp of numeracy, older striving learners do not have the building blocks of mathematics fluency to build future mathematics knowledge. As Essential Lessons build numeracy skills through their lessons, students will not only master the content, but they will also start to increase their understanding of the grade-level content taught during their traditional mathematics class.

How Essential Lessons Build Number Sense

Estimation

There is wide agreement among mathematics educators that the ability to estimate the results of computations is an essential skill. Most curriculums do not give sufficient time to estimation; therefore, it is often a skill that many students do not master in elementary school (Reys et al., 1982). Essential Lessons explicitly develop students' skills at estimating and using estimation to check the reasonableness of their calculations.

Base 10 Number Systems and Operations

Understanding place value as it applies to base 10 number systems and operations is a key element of numeracy addressed in Essential Lessons. Both the place value associated with base 10 number systems, as well as adding, subtracting, dividing, and multiplying whole numbers, fractions, and decimals, are all topics included in the lesson scope and sequence.

Research shows that place value is a very complex topic for children to grasp and is the basis for being able to do more complete base 10 number systems and operations (Schifter et al., 2016). Understanding multidigit place value is developing throughout Grades 3 and 4, and a solid understanding of place value as it applies to multidigit numbers should develop by the end of Grade 4. By the end of Grade 5, students should have a solid understanding of our base 10 place value system as it applies to all whole numbers and decimals (Foote et al., 2014). Additionally, these concepts build upon each other. For example, multiplication starts as repeated addition and builds with more complex numbers for that more conceptual understanding (Mulligan & Mitchelmore, 1997). Therefore, if a student does not gain one skill, others in later grades may not develop as well. Essential Lessons are structured to ensure each of the fundamental understanding of base 10 and operations is demonstrated.

Fractions and Decimals

Even though there is a lot of time and attention given to fraction and decimal arithmetic, many adults and children continue to need support with these concepts. This is problematic because these skills are required to access higher mathematics understanding as well as to be successful in many occupations (Hugues Lortie-Forgues et al., 2015).

A 2008 US education report noted: Difficulty with fractions (including decimals and percents) is pervasive and is a major obstacle to further progress in mathematics, including algebra. A nationally representative sample of teachers of Algebra I who were surveyed for the Panel rated students as having very poor preparation in rational numbers and operations involving fractions and decimals (Hugues Lortie-Forgues et al., 2015).

However, research shows that interventions that are specifically targeted at supporting children with overcoming difficulties with fractions and decimals can produce substantial gains in performance and understanding (Hugues Lortie-Forgues et al., 2015). One study investigated the role of students' everyday knowledge of decimals in supporting the development of their knowledge of decimals. Half the pairs worked on problems presented in familiar contexts, and half worked on problems presented without context. A comparison of pretest and posttest results revealed that students who worked on contextual problems made significantly more progress in their knowledge of decimals than did those who worked on non-contextual problems (Irwin, 2001).

Essential Lessons structure lessons in middle school age-appropriate examples, creating context that allows the learners to grasp fraction and decimal understanding.

Effective Mathematics Instruction

Integration of Skills Development with Conceptual Development

Essential Lessons allow students to build skills and conceptual development simultaneously by structuring lessons to focus on both the skills work and the conceptual, the understanding of the why that supports the computations. The NCTM *Principles to Actions* framework emphasizes the need to build procedural fluency within the context of developing conceptual understanding, therefore building procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems (NCTM, 2014), therefore integrating conceptual understanding and skill proficiency.

Based on his review of empirical research on mathematics teaching and learning, Hiebert (2003) found that effective programs focus on both conceptual understanding and skills proficiency. He concluded that “both knowing and doing” are necessary for effective learning to occur. He advises that mathematics instruction should not abandon skills development, but rather should incorporate skills development into meaning making through the construction of knowledge “while solving problems and . . . [communicating their] ideas with others.” Hiebert reported on studies comparing primary grade students who participated in a year of instruction that promoted conceptual understanding and skills development to students receiving instruction stressing only skills development. Students receiving the conceptual-plus-skills development instruction demonstrated deeper conceptual understanding and were better able to develop new procedures or modify existing ones to solve novel problems. This greater understanding appeared to improve skills development rather than detract from it (Hiebert, 2003). Noted mathematics educator Marilyn Burns agrees that understanding procedures, not just carrying them out, is essential for learning how to approach and solve problems in novel situations: We must expect and demand that students learn to understand procedures, not only perform them. When their learning is based on understanding, students won’t be incapacitated if they forget a rule or step in a rule. Only with understanding will students be prepared to apply rules correctly in new situations (Burns, 1998). NCTM President Diane Briars frames the need to integrate procedural knowledge with conceptual understanding in terms of workforce preparedness: Being prepared for the 21st-century workforce requires being able to do more than simply compute or carry out procedures. Children need conceptual understanding as well as procedural fluency, and they need to know how, why, and when to apply this knowledge to answer questions and solve problems. They need to be able to reason mathematically and communicate their reasoning effectively to others (Briars, 2014).

The three sections of each lesson in Essential Lessons build for students this progress between skills development and conceptual understanding. The Exploration builds curiosity through the investigation, inspiring the learner to explore the conceptual basis of the lesson. This also offers real-world application of the lesson so that the learner can create connections between what they are learning and their daily life. In the Application section, skill development is honed through the task and the ability to go to a second task to allow for demonstration of success of the skill. The Retry ensures that the student has demonstrated the skill before moving on to the next lesson.

Teaching for Meaning

Essential Lessons focus on helping students construct meaning as they work to solve problems throughout the lesson. Sfard notes that the NCTM Standards call for instructional shifts needed to support students in their construction of meaning:

- Toward logic and mathematical evidence as verification—away from the teacher as the sole authority for right answers
- Toward mathematical reasoning—away from merely memorizing procedures
- Toward connecting mathematics, its ideas, and its applications—away from treating mathematics as a body of isolated concepts and procedures (Sfard, 2003).

Similarly, in his review of empirical research on mathematics teaching and learning, Hiebert (2003) found that traditional mathematics instruction focused on explaining, demonstrating, and discussing basic skills and procedures, along with student practice of those same skills and procedures. In contrast, alternative mathematics programs that have the goal of more meaningful learning build "directly on students' entry knowledge," provide opportunities to solve problems that "require some creative work by students and some practice of already learned skills," focus on comparison of multiple methods for solving problems, and ask students to explain and justify their solutions.

In the Exploration section of a lesson, students generally solve problems before developing a formal rule. They first explore tasks on their own, then use their experience in exploration to articulate a "key idea," rather than simply being told a rule. This allows for them to make meaning as they are going through the lesson, which also deepens their conceptual understanding.

Impacts of Teaching for Meaning and Skills Development

In their examination of US mathematics education from Grade K to graduate school, the National Research Council (NRC), in conjunction with the Committee on the Mathematical Sciences, embarked on a multiyear project to identify strengths and weaknesses in the teaching of mathematics. In their report, they concluded that the traditional "passive" pedagogy ends up reinforcing mastery without understanding: Students simply do not retain for long what they learn by imitation from lectures, worksheets, or routine homework. Presentation and repetition help students do well on standardized tests and lower-order skills, but they are generally ineffective as teaching strategies for long-term learning, for higher-order thinking, and for versatile problem solving (NRC, 1989).

The NRC report points to research that shows that allowing students to construct their own understanding results in more effective learning: Educational research offers compelling evidence that students learn mathematics well only when they construct their own mathematical understanding. To understand what they learn, they must enact for themselves verbs that permeate the mathematics curriculum: "examine," "represent," "transform," "solve," "apply," "prove," and "communicate" (NRC, 1989). Similarly, Hiebert's (2003) review of mathematics teaching practices and learning revealed that with traditional pedagogy, students tend to learn the simplest knowledge and basic skills without much depth or conceptual understanding. Evidence for this was poor performance on items that require students to extend these skills, reason about them, or explain why they work. By contrast, Hiebert (2003) also found that with alternative programs that teach meaning, students constructed a deeper understanding of the concepts that underlie the procedures. This understanding showed itself in a variety of ways, including students' ability to invent new procedures or modify old ones to solve new problems.

CONCLUSION

Essential Lessons address the critical domains specified in the NCTM and CCSS and follow key recommendations from these initiatives, founded in research and expert opinion:

- **Promotion** of meaning making in mathematics
- **Integration** of skills development and conceptual development
- **Providing problem-solving contexts** for learning mathematics

The goal of Essential Lessons is to ensure older striving middle school learners are able to fill in gaps in their mathematical understanding so they can be successful in engaging with grade-level middle school content. In order to meet this goal, Essential Lessons focus on the most critical domains, starting with number sense and numeracy, specifically magnitude, estimation, comparison problems, base 10 number system and operations, and fractions/decimals. As explained above, numeracy is the most foundational skill to access higher-level mathematics concepts. Learning is based on understanding, so that students won't be incapacitated if they forget a rule or a step in a rule.

Essential Lessons encourage students to persist in their efforts to understand mathematics concepts and solve problems. Essential Lessons activities offer students the opportunity to engage with challenging mathematics problems in a deep way, which involves the possibility of initial struggle. If students are not successful with a problem at first, they are given a new set of more simple numbers to ensure they conceptually understand. This helps students develop the sense that additional effort will lead to success.

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