Research Base for *i-Ready Personalized Instruction* for Mathematics

Table of Contents

Executive Summary

Research on mathematics instruction provides a solid foundation of practices that help students gain proficiency in the subject. Curriculum for K–8 that supports teachers in translating research and expert recommendations into practice can make significant headway in improving mathematics proficiency for all students.

Standards-based instruction. Recommendations from the National Council of Teachers of Mathematics (NCTM) and the Common Core State Standards (CCSS) Initiative include the following:

- Curriculum content in the elementary grades should address number and operations, algebraic thinking, measurement, data analysis, and geometry.
- Instruction should promote sense making and understanding of mathematical concepts and procedures in the context of meaningful problem solving.
- Sense making requires encouragement of and guidance on reasoning about relationships among mathematics concepts and procedures and on recognizing patterns and structure within problem situations so students come to see mathematics as a system of interrelated ideas.
- Instruction should support student interpretation and development of simplified models of real-life problems, represented in multiple ways—graphically, symbolically, and verbally.
- Instructional activities should encourage meaningful communication with others about mathematical arguments, evidence, and conclusions.
- Instructional activities should encourage and provide support for perseverance and productive struggle when attempting to solve challenging mathematics problems (CCSS Writing Team, 2011–2018; CCSS Initiative, 2019; NCTM, 2000, 2014; NGACBP & CCSSO, 2010; NRC, 2001).

Meaning making in mathematics. Research-based recommendations about making meaning in mathematics include the following:

- Instructional activities should be planned and sequenced so students can extract mathematical concepts early on from concrete and action-based experience. Then, as students develop basic concepts, instructional activities should involve exploration of how these basic concepts are related (Sfard, 2003).
- Mathematics should be taught in the context of problem-solving activities (Burns, 2015; Lesh & Zawojewski, 2007; Hiebert, 2003; NCTM, 2000; Sfard, 2003).
- Students should be active participants in mathematics learning who engage in instructional activities that involve examining, representing, transforming, solving, applying, proving, and communicating about mathematical concepts and procedures (Hiebert, 2003; NRC, 1989).

Developing conceptual, factual, and procedural knowledge. Research-based recommendations about the relationships among conceptual, factual, and procedural knowledge include the following:

- Educators should take into account children's informal entry knowledge and build on it, while also considering individual differences (Baroody & Ginsburg, 1986; Hiebert, 2003).
- Instructional activities should feature mathematics problems that call for students to invent solutions based on their prior knowledge and understanding, while also providing practice of mathematics skills they have previously learned (Hiebert, 2003; Sfard, 2003).
- Students should be guided to understand the thinking behind mathematics procedures before over-practicing them (Baroody & Ginsburg, 1986; Hiebert, 2003).

Problem solving, modeling, and representation. Research-based recommendations about incorporating problem solving in mathematics education include the following:

- Problem solving should provide a context for developing both conceptual understanding and procedural knowledge (Sfard, 2003).
- Instructional activities should often involve students in solving problems that require them to both think inventively and practice the skills they have already learned. The problems should require students to draw upon the system of mathematical ideas they already understand well (Hiebert, 2003; Sfard, 2003).
- Problem-solving activities should strike a balance in terms of the level of challenge—offering enough challenge to stimulate further development of conceptual understanding, but not so much challenge as to cause frustration. In this regard, instruction should take into account individual differences among students (Sfard, 2003).
- Problem-solving activities should include representation of problems in mathematical models, such as diagrams, graphs, and equations (Lesh & Zawojewski, 2007; NCTM, 2000, 2014). Over time, problem solving should help students develop representational fluency among different representational forms (Lesh & Zawojewski, 2007).
- Mathematics instruction should encourage analysis of multiple methods of solving problems and comparison of methods (Hiebert, 2003).
- Educators should encourage students to provide explanations of their method(s) of solving mathematics problems (Burns, 2015; Hiebert, 2003).

Growth mindset. Research suggests that instruction should encourage persistence of effort rather than reinforcing the idea that intelligence and ability are fixed. Teachers and instructional tools should focus student praise on the steps students are taking to master learning material rather than on their innate intelligence (Blackwell, Trzesniewski, & Dweck, 2007; Dweck, 2015).

How *i-Ready* aligns with the research. *i-Ready Personalized Instruction* for Mathematics addresses all of the critical domains specified in the NCTM and CCSS—Number and Operations, Algebra and Algebraic Thinking, Measurement and Data, and Geometry—and follows key recommendations from these initiatives, research, and expert opinions.

- *i-Ready Personalized Instruction* for Mathematics lessons are designed to help students construct meaning around mathematics concepts and principles, basic mathematics facts, and procedures—and the structural relationships among these.
- The adaptive *i-Ready Diagnostic* assessment places each student in a personalized learning path through online lessons. This helps ensure that the lessons build on each student's prior knowledge and that each student is working at an appropriate level of challenge.
- Lessons are designed around problem-solving challenges that call for a combination of invention and practice of previously learned procedures. Students are encouraged to try out multiple strategies to solve a problem. The mathematics problems are significant in the sense that they require students to use their prior knowledge about the system of mathematical concepts. Problem scenarios are designed to be meaningful and relevant to students as well as culturally and linguistically responsive.
- *i-Ready Personalized Instruction* for Mathematics problem-solving activities involve representation of problems in mathematical models. Instruction lessons present students with tools to use in modeling the problem at hand. If students struggle in their use of representational tools, the system provides scaffolded support.
- *i-Ready Personalized Instruction* for Mathematics lessons balance opportunities for problem solving with practice of previously learned skills. Lessons include a mix of open and guided explorations of challenging problems to solve, plus independent practice of previously learned concepts and procedures.
- *i-Ready* provides teachers with lesson plans in the Tools for Instruction—to be used independently of the online lessons—that encourage whole class or small group conversation about students' various strategies for solving mathematics problems.
- *i-Ready* encourages students to persist in their efforts to understand mathematics concepts and solve problems. When students don't succeed with a challenging problem, scaffolded feedback suggests strategies for successfully solving the problem and encourages them to try again. Progress monitoring screens for younger students (Grades K–2) praise them for their efforts and perseverance, and the student dashboard provides students across the K–8 grade span with a record of their effort over time.

Introduction

The ability to solve problems by applying mathematical concepts, facts, and procedures is critical to success in school and beyond, yet many students lack sufficient mathematics skills for such success. The National Assessment of Educational Progress (NAEP) determined that only 41% of fourth grade students and only about 34% of eighth grade students performed at or above the "Proficient" level in mathematics in 2019 (US Department of Education, 2020). Of the high school graduates who took the ACT exam in 2019, a majority (61%) did not meet ACT's college benchmarks for mathematics (ACT, 2019).

The good news is that research on mathematics instruction provides a solid foundation of practices that help students gain proficiency in the subject. Curriculum that supports teachers in translating research and expert recommendations into practice can make significant headway in improving mathematics proficiency for all students.

i-Ready: Connecting Research to Practice

i-Ready combines diagnostic assessment, engaging online lessons, and downloadable resources for classroom instruction to support teachers in providing differentiated instruction that meets the needs of each learner.

i-Ready Diagnostic is an adaptive assessment that offers a detailed picture of student performance and growth across the school year while also providing teachers with actionable insight into student needs. By assessing a broad range of skills and adapting the assessment items to student responses, *i-Ready Diagnostic* pinpoints each student's ability level, identifies the skill areas each student needs to develop in order to accelerate growth, and delivers a personalized learning path.

Online lessons in *i-Ready Personalized Instruction* for Mathematics tap into the rich data from *i-Ready Diagnostic* and deliver tailored instruction that meets students where they are. Students receive opportunities to explore mathematics concepts, explicit mathematics instruction, systematic practice, and scaffolded feedback that encourages them as they develop new skills. Lessons are tested extensively with younger students and older struggling learners, ensuring that *i-Ready Personalized Instruction* for Mathematics is engaging and effective for students of all abilities and ages. Instructional reports allow teachers to monitor how students are responding to this instruction and point them to downloadable lesson plans and other instruction resources (e.g., Tools for Instruction) that teachers can use for remediation and reteaching.

The lessons in *i-Ready Personalized Instruction* for Mathematics supplement classroom mathematics instruction and bolster the skills of on-grade level, advanced, and struggling learners. Instruction focuses on a broad range of domains that research tells us are important in order to develop mathematics proficiency—Number and Operations, Algebra and Algebraic Thinking, Measurement and Data, and Geometry. Lessons focused on each of these domains are designed to reflect research and expert opinion on effective mathematics instruction.

Research on *i-Ready* shows that it is an effective resource for accelerating student growth and progress toward mathematics proficiency, meeting Level 3 criteria for the Every Student Succeeds Act (ESSA).

In a comprehensive study conducted by Curriculum Associates, *i-Ready Diagnostic* data from more than four million students indicated that on average, students across Grades K–8 using *i-Ready Personalized Instruction* experienced score gains in mathematics that were 38% greater than students who did not use *i-Ready Personalized Instruction* (Curriculum Associates, 2017). Research also evaluated the impact for subgroups and found similar results, with non-Caucasian students, students with disabilities, students with socioeconomic disadvantages, and English Learners who experienced *i-Ready Personalized Instruction* demonstrating greater gains in mathematics than students in these subgroups who did not receive *i-Ready Personalized Instruction* (Curriculum Associates, 2017). These results indicate that *i-Ready Personalized Instruction* for Mathematics is an effective system for accelerating student growth and progress toward mathematics proficiency.

Curriculum Associates notes that *i-Ready*'s effectiveness is due, in part, to its research-based design. This paper presents research and expert opinion on several aspects of effective mathematics instruction and explains how *i-Ready Personalized Instruction* for Mathematics aligns to this research.

Standards-Based Mathematics Instruction

Two sets of curriculum standards have been largely responsible for driving reform in mathematics instruction across the United States: the NCTM Principles and Standards for School Mathematics (NCTM, 2000) and the CCSS for Mathematics (NGACBP & CCSSO, 2010). Most states' curriculum standards have been informed by one or both of these works, including states that have not adopted the CCSS and states that have revised their standards after having adopted the CCSS.

In 2014, the NCTM published *Principles to Actions: Ensuring Mathematical Success for All*, which recommends actions for mathematics educators, school and district administrators, other education leaders, and policymakers, reflecting the CCSS and "more than a decade of experience and new research evidence about excellent mathematics programs" (NCTM, 2014).

Together, these and related documents provide research-informed guidance on the "what" and "how" of high-quality mathematics instruction.

NCTM's Principles and Standards for School Mathematics

The NCTM Principles and Standards were developed to provide a "comprehensive and coherent set of learning goals for mathematics" as a guide for "teachers, education leaders, and policymakers" and for "the development of curriculum frameworks, assessments, and instructional materials" (NCTM, 2000). The Principles and Standards include five Content Standards: Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability.

NCTM Process Standards

The Principles and Standards also included five Process Standards:

- **Problem Solving**—providing "frequent opportunities to formulate, grapple with, and solve complex problems" and encouraging students to reflect on the thinking strategies they use, with an eye toward applying and adapting these strategies "to other problems and in other contexts"
- **Reasoning and Proof**—developing students' reasoning and analytical thinking skills in the context of "real-world and mathematical situations" by having them recognize "patterns [and] structure" within such situations, make and explore "mathematical conjectures," and "develop and evaluate mathematical arguments and proofs"
- **Communication**—challenging students "to communicate the results of their thinking to others" using "mathematical arguments and rationales"
- **Connections**—presenting "mathematics [as] an integrated field of study" by stressing "the interrelatedness of mathematical ideas"
- **Representations**—guiding students to create and interpret mathematical ideas "represented in a variety of ways," including pictures, concrete materials, tables, graphs, and symbols (NCTM, 2000)

In 2003, the NCTM published *A Research Companion to Principles and Standards for School Mathematics* to present "perspectives from theory, research, and practice" related to the NCTM Standards (Kilpatrick, Martin, & Schifter, 2003). Selected chapters from this work are reviewed in this paper.

CCSS for Mathematics

The CCSS initiative was led by the NGACBP and the CCSSO with the intent of developing "consistent, real-world learning goals . . . to ensure all students, regardless of where they live, are graduating high school prepared for college, career, and life" (CCSS Initiative, 2019).

The CCSS for Mathematics were informed of the "experience of teachers, content experts, states, and leading thinkers" and "feedback from the public" by existing high-quality state standards (CCSS Initiative, 2019). Each Common Core mathematics standard is a statement about "what students should understand and be able to do." Standards are organized in "clusters" of related standards, and related clusters are organized in "domains" at each K–8 grade level (NGACBP & CCSSO, 2010). The domains for Grades K–8 include:

- Counting and Cardinality (Grade K)
- Operations and Algebraic Thinking (Grades K–5)
- Number and Operations in Base Ten (Grades K–5)
- Number and Operations—Fractions (Grades 3–5)
- Measurement and Data (Grades K–5)
- Geometry (Grades K–8)
- Ratios and Proportional Relationships (Grades 6–7)
- The Number System (Grades 6–8)
- Expressions and Equations (Grades 6–8)
- Statistics and Probability (Grades 6–8)
- Functions (Grade 8)

To provide additional support to educators, the CCSS Writing team has disseminated a set of Progressions for the CCSS in Mathematics (2011–2018). Each Progressions document presents a grade-by-grade sequence of knowledge and skills students should develop, which includes guidance on teaching for conceptual understanding.

CCSS for Mathematical Practice

The CCSS for Mathematics include Standards for Mathematical Practice that specify different types of "expertise that mathematics educators at all levels should seek to develop in their students" (NGACBP & CCSSO, 2010). Foundational sources in the development of these practice standards include the NCTM Process Standards (summarized on page 9) and mathematics proficiency strands presented in the National Research Council's (2001) report *Adding It Up* ("adaptive reasoning, strategic competence, conceptual understanding . . ., procedural fluency . . ., and productive disposition").

The eight Standards for Mathematical Practice are as follows:

1. **"Make sense of problems and persevere in solving them"**—students' ability to explain "the

meaning of a problem" and ways to get started toward a solution; analyze "givens, constraints, relationships, and goals;" make "conjectures" and plan "a solution pathway;" "consider analogous problems . . . in order to gain insight into [finding a] solution;" and "monitor and evaluate their progress" in terms of whether their approach makes sense and "change course if necessary"

2. **"Reason abstractly and quantitatively"**—students' ability to make sense of quantitative "relationships in problem situations" in order to create "a coherent representation of the problem at hand." This includes *decontextualizing*—moving from understanding the problem situation to symbolic representation and manipulation—and *contextualizing*—pausing along the way to consider the meaning of the symbols relative to the problem situation and whether the symbolic manipulations make sense.

3. **"Construct viable arguments and critique the reasoning of others"**—students' ability to construct logical arguments using "stated assumptions, definitions, and previously established results;" "make conjectures" and test them through "a logical progression of statements;" "analyze situations" in terms of example cases and counterexamples; make arguments that are plausible given the problem context; "justify [and communicate] their conclusions;" engage with others about arguments against their conclusions; compare competing plausible arguments

4. **"Model with mathematics"**—students' ability to apply their mathematical knowledge to solve real-life problems by "making assumptions and approximations to simplify a complicated situation" and revising as needed later on; representing relationships between "quantities in a practical situation . . . using such tools as diagrams, . . . tables, graphs, flowcharts and formulas;" analyzing "those relationships mathematically to draw conclusions;" interpreting the results in the problem context to consider if the results make sense or if the mathematical model needs improvement

5. **"Use appropriate tools strategically"**—students' ability to consider and make good decisions about "using the available tools when solving a mathematical problem," such as pencil and paper, concrete models, other physical tools for measurement, and technology-based mathematics tools and systems

6. **"Attend to precision"**—students' ability to "communicate precisely to others" about mathematical situations, problems, and solutions, including use of "clear definitions;" explaining "the meaning of the symbols they choose;" "specifying units of measure;" accurate labeling of graphic representations. This standard also calls for students to "calculate accurately and efficiently" and "express numerical answers with a degree of precision appropriate for the problem context."

7. **"Look for and make use of structure"**—students' habit and ability to examine a mathematical situation closely to "discern a pattern," including commonalities between related expressions or phenomena and ways of breaking a new, more complex problem into simpler, more familiar problems

8. **"Look for and express regularity in repeated reasoning"**—students' habit and ability to explore whether "calculations are repeated and look both for general methods and for shortcuts" while "continually evaluat[ing] the reasonableness of their intermediate results" (NGACBP & CCSSO, 2010)

The authors of the CCSS for Mathematics recommend that "designers of curricula, assessments, and professional development . . . attend to the need to connect the mathematical practices to mathematical content in mathematics instruction." They advise that standards that begin with the verb "understand" tend to be "especially good opportunities to connect the practices to the content."

NCTM's Principles to Actions

In its 2014 publication, *Principles to Actions: Ensuring Mathematical Success for All*, the NCTM offered "research-informed" recommendations for educator implementation of the CCSS for Mathematics, which the NCTM called "Mathematics Teaching Practices":

- **"Establish mathematics goals to focus learning."**
- **"Implement tasks that promote reasoning and problem solving."**
- **"Use and connect mathematical representations."**
- **"Facilitate meaningful mathematical discourse."**
- **"Pose purposeful questions** . . . to assess and advance students' reasoning and sense making about important mathematical ideas and relationships."
- **"Build procedural fluency from conceptual understanding."**
- **"Support productive struggle in learning mathematics."**
- **"Elicit and use evidence of student thinking** . . . to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning." (NCTM, 2014)

Common Recommendations across the NCTM and CCSS Mathematics Sources

The work of the NCTM and the CCSS Initiative's Mathematics team can be seen as a back-and-forth, iterative process. The NCTM's *Principles and Standards for School Mathematics* (2000) was a foundational source for the authors of the CCSS for Mathematics (2010). More recently, the NCTM's *Principles to Actions* (2014) provided guidance to the education community on research-based best teaching practices for implementation of the CCSS.

Analysis across the work of the NCTM and the CCSS Initiative suggests the following common standards-based recommendations:

- Curriculum content in the elementary grades should address number and operations, algebraic thinking as preparation for formal algebra in later grades, measurement, data analysis as preparation for the study of probability in later grades, and geometry.
- Instruction should promote sense making and understanding of mathematical concepts and procedures in the context of meaningful problem solving.
- Sense making requires encouragement of and guidance on reasoning about relationships among mathematics concepts and procedures as well as on recognizing patterns and structure within problem situations, so students come to see mathematics as a system of interrelated ideas.
- Instruction should support student interpretation and development of simplified models of real-life problems, represented in multiple ways—graphically, symbolically, and verbally.

- Instructional activities should encourage meaningful communication with others about mathematical arguments, evidence, and conclusions.
- Instructional activities should encourage and provide support for perseverance and productive struggle when attempting to solve challenging mathematics problems.

Research on Meaning Making in Mathematics

Research and expert opinion support the conclusion that the act of meaning making is essential for successful mathematics learning to take place, and helping students see structural relationships is key to meaning making. There is consensus among researchers that students can better construct meaning in the context of mathematical problem solving. Successful mathematics instruction has shifted toward supporting more meaningful learning by building on students' entry knowledge, providing opportunities for invention and practice, examining multiple problem-solving methods, and calling on students to provide explanations of methods and why they work. Skills development remains important, but it needs to be incorporated into students' construction of knowledge.

Learning as Meaning Making

In her analysis of the research connected to the NCTM framework, Sfard (2003) summarizes the work of researchers and learning theorists indicating the importance of meaning making in driving learning:

> *The . . . need for meaning and the need to understand ourselves and the world around us have come to be widely recognized as the basic driving force behind our intellectual activities. . . . The need for meaning . . . [is] what motivates and guides our learning.*

Sfard (2003) and other researchers she cites reject the idea of the learner "passively absorbing externally generated experiences" and the notion that the human mind is a preprogrammed, "predesigned product of nature." Rather, it is the act of *constructing meaning* that results in successful learning.

> *. . . the meaning of ideas is constructed anew each time anyone learns these ideas . . . the learner is given the . . . exciting and responsible role of autonomous meaning builder.*

Understanding Means "Seeing" Structure

Structure is essential for developing understanding and making meaning. Prominent theorists have weighed in on this important relationship between seeing structural relationships and developing understanding: "The idea of understanding [is] almost tantamount to seeing relations . . ." (Sfard, 2003; Vygotsky, 1962, 1987; Brownell, 1935; Skemp, 1976). Sfard (2003) cites Skemp's (1976) distinction between "know[ing] rules without reasons" and "knowing both what to do and why." In mathematics, knowing why requires understanding the structural relationships among the facts, concepts, and procedures.

Sfard traces the many levels of structure in mathematical learning for meaning:

> *. . . first the structures that can be extracted directly from concrete things and actions and that constitute the basic mathematical concepts . . . then the structures obtained through investigation of relations between these most fundamental mathematical structures, and so on. . . . If understanding means seeing structure, then the well-organized connections among concepts already learned and those that the students are yet to learn must never disappear.*

The authors of the NCTM Standards link the standards to mathematical structures and support using problem scenarios as relevant contexts for "extracting" mathematical structure.

> *The idea that mathematical conceptions are created from objects, events, and relations. . . is promoted throughout. Also, the need to appreciate the overall structure of mathematics gets a good deal of attention* (Sfard, 2003).

Teaching Mathematics for Meaning in a Problem-Solving Context

There is general agreement among researchers, mathematicians, and the NCTM Standards about the value of having students construct meaning in the context of mathematical problem solving (Burns, 2015; Lesh & Zawojewski, 2007; Hiebert, 2003; NCTM, 2000; Sfard, 2003). (For more detail about problem solving in mathematics instruction, **see Problem Solving, Modeling, and Representation** later on in this paper.)

Comparing Traditional Mathematics Pedagogy to Alternative Teaching for Meaning

Sfard notes that the NCTM Standards call for instructional shifts needed to support students in their construction of meaning:

- *"Toward logic and mathematical evidence as verification—away from the teacher as the sole authority for right answers"*
- *"Toward mathematical reasoning—away from merely memorizing procedures"*
- *"Toward conjecturing, inventing, and problem solving—away from an emphasis on mechanistic answer-finding"*
- *"Toward connecting mathematics, its ideas, and its applications—away from treating mathematics as a body of isolated concepts and procedures"* (Sfard, 2003)

Similarly, in his review of empirical research on mathematics teaching and learning, Hiebert (2003) found that traditional mathematics instruction focused on explaining, demonstrating, and discussing basic skills and procedures, along with student practice of those same skills and procedures. In contrast, alternative mathematics programs that have the goal of more meaningful learning build "directly on students' entry knowledge," provide opportunities to solve problems that "require some creative work by students and some practice of already learned skills," focus on comparison of multiple methods for solving problems, and ask students to explain and justify their solutions.

Comparing Impacts of Traditional Mathematics Pedagogy and Alternative Teaching for Meaning

In their examination of US mathematics education from kindergarten to graduate school, the National Research Council (NRC), in conjunction with the Committee on the Mathematical Sciences, embarked on a multiyear project to identify strengths and weaknesses in the teaching of mathematics.

In their report, they concluded that the traditional "passive" pedagogy ends up reinforcing mastery without understanding:

> *Students simply do not retain for long what they learn by imitation from lectures, worksheets, or routine homework. Presentation and repetition help students do well on standardized tests and lower-order skills, but they are generally ineffective as teaching strategies for long-term learning, for higher-order thinking, and for versatile problem solving* (NRC, 1989).

The NRC report points to research that shows that allowing students to construct their own understanding results in more effective learning:

> *Educational research offers compelling evidence that students learn mathematics well only when they construct their own mathematical understanding. To understand what they learn, they must enact for themselves verbs that permeate the mathematics curriculum: "examine," "represent," "transform," "solve," "apply," "prove," "communicate"* (NRC, 1989).

Similarly, Hiebert's (2003) review of mathematics teaching practices and learning revealed that with traditional pedagogy, students tend to learn the simplest knowledge and basic skills "without much depth or conceptual understanding." Evidence for this was poor performance on "items that require students to extend these skills, reason about them, or explain why they work."

In contrast, Hiebert (2003) found that with alternative programs that teach for meaning, students:

> *. . . constructed a deeper understanding of the concepts . . . that underlie the procedures. This understanding showed itself in a variety of ways, including students' ability to invent new procedures or modify old ones to solve new problems.*

Balancing Meaning Making and Skills Development

The NCTM Standards and multiple research efforts support the integration of teaching for conceptual understanding and development of procedural skills (Baroody & Ginsburg, 1986; Hiebert, 2003; NCTM, 2000; Sfard, 2003). (For more detail about this integration in mathematics instruction, **see Conceptual, Factual, and Procedural Knowledge** later on in this paper.)

Research-Based Recommendations for Mathematics Instruction

Recommendations based on the research sources reviewed in this section about making meaning in mathematics include the following:

- Instructional activities should be planned and sequenced so students can, early on, extract mathematical concepts from experience with "concrete things and actions." Then as students develop basic concepts, instructional activities should involve "investigation of relations" between these basic concepts (Sfard, 2003).
- Mathematics should be taught in the context of problem-solving activities (Burns, 2015; Lesh & Zawojewski, 2007; Hiebert, 2003; NCTM, 2000; Sfard, 2003).

- Students should be active participants in mathematics learning who are encouraged to "invent new procedures or modify old ones to solve new problems" (Hiebert, 2003). Instructional activities should involve students in examining, representing, transforming, solving, applying, proving, and communicating about mathematical concepts and procedures (NRC, 1989).

For additional research-based recommendations, see the later sections of this paper.

Research on Developing Conceptual, Factual, and Procedural Knowledge

According to the NCTM's *Principles and Standards for School Mathematics*, learning mathematics with understanding requires integration of three types of knowledge: conceptual knowledge, factual knowledge, and procedural proficiency (NCTM, 2000). Research and expert opinion suggest that a teaching–learning environment that emphasizes and supports conceptual understanding can also strengthen factual knowledge and procedural proficiency.

Evidence from Early Arithmetic Learning

Baroody and Ginsburg (1986) investigated young children's development of informal understanding of arithmetic concepts and procedures. They distinguish between "meaningful knowledge"—implicit or explicit understanding of mathematical concepts and principles—and "mechanical knowledge"—"knowledge of facts (specific associations) and procedural knowledge (rules and algorithms)." These researchers characterize the relationship between meaningful and mechanical knowledge as "a complex one."

Example. Baroody and Ginsburg (1986) offer an example of early conceptual understanding leading to a progression of procedural strategies, which leads to deeper conceptual understanding. They found that children go through a sequence of concrete and mental counting and addition strategies. For example, the earliest addition strategy is typically counting all concrete objects. After a sequence of child-developed procedures, they eventually count on from the larger term of an addition problem (e.g., for 2 + 4, they start with 4, then count 5 [+ 1], then 6 [+ 2]). These researchers identified several factors that can guide a child's choice of procedure, including "the semantic structure of addition word problems," "the need to reduce the load on working memory," and "problem size." Eventually, after multiple attempts at computing sums and seeing the results, they experience a conceptual "breakthrough": understanding of the principle of *commutativity*—that order of the terms in addition does not matter (e.g., 2 + 6 = 6 + 2).

Competing models of basic mathematics fact acquisition. Baroody and Ginsburg distinguish between two theoretical models of basic arithmetic fact learning: "associative learning," which assumes that each mathematics fact is stored separately in "the mental arithmetic table," and a semantic model, which assumes that "factual, conceptual, and procedural knowledge form an integrated unity" and that "data and propositional processes interact to *create answers* efficiently." These experts make the case that a semantic model of basic arithmetic-fact learning better fits the available evidence than does an associative-learning model.

Effective Mathematics Instruction Integrates Skills Development with Conceptual Development

The NCTM *Principles to Actions* framework emphasizes the need to build procedural fluency within the context of developing conceptual understanding:

> ***Build procedural fluency from conceptual understanding.*** *Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems* (NCTM, 2014).

Integrating conceptual understanding and skill proficiency. Based on his review of empirical research on mathematics teaching and learning, Hiebert (2003) found that effective programs focus on both conceptual understanding and skills proficiency. He concluded that "both knowing *and* doing" are necessary for effective learning to occur. He advises that mathematics instruction should not abandon skills development, but rather should incorporate skills development into meaning making through the construction of knowledge "while solving problems and . . . [communicating their] ideas with others."

Hiebert reported on studies comparing primary grade students who participated in a year of instruction that promoted conceptual understanding *and* skills development to students receiving instruction stressing only skills development. Students receiving the conceptual-plus-skills-development instruction demonstrated deeper conceptual understanding and were better able to develop new procedures or modify existing ones to solve novel problems. This greater understanding appeared to improve skills development rather than detract from it (Hiebert, 2003).

Noted mathematics educator Marilyn Burns agrees that understanding procedures, not just carrying them out, is essential for learning how to approach and solve problems in novel situations:

> *We must expect and demand that students learn to understand procedures, not only perform them. When their learning is based on understanding, students won't be incapacitated if they forget a rule or step in a rule. Only with understanding will students be prepared to apply rules correctly in new situations* (Burns, 1998).

NCTM President Diane Briars frames the need to integrate procedural knowledge with conceptual understanding in terms of workforce preparedness:

> *. . . being prepared for the 21st-century workforce requires being able to do more than simply compute or carry out procedures. Children need conceptual understanding as well as procedural fluency, and they need to know how, why, and when to apply this knowledge to answer questions and solve problems. They need to be able to reason mathematically and communicate their reasoning effectively to others* (Briars, 2014).

Attending to recurring procedures. Based on Sfard's (2003) review of research and theory related to the NCTM Standards, along with critiques of the Standards, she concluded that in addition to focusing on meaning and structure, mathematics instruction needs to attend to "repetitive, well-defined actions"—in other words, recurring procedures. She argues that deep understanding of mathematics involves the study of repeatable actions. Learners must become sufficiently familiar with recurring procedures to *reify* them—that is, "to be able to think and speak about the process in ways in which we think and speak about an object."

Sfard asserts that the NCTM Standards don't call for the abandonment of basic skills but rather "require that the skills be developed in new, more 'natural' ways." However, she warns against a tendency for some educators "to interpret [the Standards] as denying the [basic] skills any real importance."

Research-Based Recommendations for Mathematics Instruction

Recommendations based on the research sources reviewed in this section about the relationships among conceptual, factual, and procedural knowledge include the following:

- Educators should take into account children's informal entry knowledge and build on it, while also considering individual differences (Baroody & Ginsburg, 1986; Hiebert, 2003).
- Instructional activities should provide "opportunities for both invention and practice"—often featuring problem solving that calls for "creative work by students" and "practice of already learned skills" (Hiebert, 2003; Sfard, 2003).
- Educators should discourage "blind procedure following and answer producing" (Baroody & Ginsburg, 1986).
- More specifically, mathematics instruction should avoid introducing "formal symbolism too quickly" and requiring students to "learn in a lockstep manner" (Baroody & Ginsburg, 1986). Similarly, educators should avoid over-practice of procedures before students understand them, as "it is more difficult to make sense of them later" (Hiebert, 2003).

Research on Problem Solving, Modeling, and Representation

As noted in the preceding sections of this paper, a substantial body of research shows that problem solving is a fundamental component of effective instruction in mathematics at all grades. Exposing students to tasks that promote problem solving, reasoning, and deeper understanding provides the context in which mathematics fact memorization and procedural knowledge development can flourish and be meaningfully applied. Research also shows that for problem solving to be effective, it must provide students with the right amount of challenge and significance for the student. Effective mathematics instruction also involves students in the construction and interpretation of mathematical models—graphic and symbolic representations of problem situations used as tools in problem solving.

Problem Solving in a Mathematical Context

Lesh and Zawojewski (2007) piloted a comprehensive review of the research that has been conducted on the role of problem solving and modeling in the learning of mathematics. Based on recent research, they define problem solving in mathematics as:

> *. . . the process of interpreting a situation mathematically, which usually involves several iterative cycles of expressing, testing, and revising mathematical interpretations—and of sorting out, integrating, modifying, revising, or refining clusters of mathematical concepts from various topics within and beyond mathematics*

Rather than viewing problem solving as a simple search for a procedure to move from the "givens" to the "goals" of a problem, Lesh and Zawojewski found that problem solving should be viewed as iterative cycles of deeper understanding of the "givens" and "goals" (Lesh & Zawojewski, 2007).

Problem Solving is one of five Process Standards in the NCTM's *Principles and Standards for School Mathematics* (2000) and one of eight fundamental mathematical practices explained in NCTM's more recent work, *Principles to Actions* (2014). The latter work calls out the role of tasks that "promote mathematical reasoning and problem solving" and that allow for multiple points of entry and the use of "varied solution strategies" (NCTM, 2014).

In the fourth edition of Burns's seminal work *About Teaching Mathematics* (2015), she underlines the importance of the thinking and reasoning skills involved in problem solving and that these:

> *. . . should be at the forefront of instruction. This means that students should do more than produce correct answers. . . . Students [need] to make sense of a problem that require[s] them to reason numerically, construct a viable argument, communicate their ideas, and attend to the precision of their solution.*

Challenge and Significance

To be effective, problem-solving lessons used in mathematics instruction need to have the appropriate amount of challenge. If they are too easy or too hard, learning is much less likely to occur (Sfard, 2003). Sfard ties NCTM's call for matching problem-solving lessons to the learner's maturity and prior experience to Zygotsky's work on keeping learners within their Zone of Proximal Development (ZPD) (Sfard, 2003).

For Sfard, motivating a student's learning in mathematics requires consideration of both the difficulty level of the problem and the significance of the problem-solving situation in relation to "the system of concepts" that the learner already understands well. She asserts that this connection between learning and significant relationship to prior knowledge and understanding is supported by the work of Piaget and Vygotsky. Piaget defines learning as "enriching and reorganizing existing mental schemes," whereas Vygotsky views knowledge as developing "through our constant dissatisfaction and incessant 'reworking' of what we already know" (Sfard, 2003).

Linking Problem Solving and Modeling

The NCTM *Principles to Actions* framework calls for mathematical representations, or models, to be used as tools for problem solving:

> ***Use and connect mathematical representations.*** *Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving* (NCTM, 2014).

Mathematical modeling involves using mathematical structures such as graphs, diagrams, and equations to represent authentic real-world situations. These models or representations provide an abstract and simplified representation of the essential characteristics of a problem in order to facilitate solving it.

In the earlier *Principles and Standards for School Mathematics*, the NCTM describes modeling as taking place in a series of stages:

> *Modeling involves identifying and selecting relevant features of a real-world situation, representing those features symbolically, analyzing and reasoning about the model and the characteristics of the situation, and considering the accuracy and limitations of the model* (NCTM, 2000).

In their research review, Lesh and Zawojewski (2007) draw a strong link between problem solving and developing mathematical models. They assert that students need to interpret problem situations mathematically in order to develop deep understanding of mathematical concepts, and such interpretation involves developing models.

Lesh and Zawojewski (2007) describe a process of learning mathematics through problem solving by creating, refining, and/or adapting interpretations embedded in mathematical models. These researchers contrast this approach with traditional instruction, in which problem-solving strategies are taught after procedures have been learned and authentic "real-world" problems are only taught in the final stages of instruction.

The Role of Representational Fluency in Learning Mathematics

Research indicates that the development of *representational fluency* is essential for real-world applications. Representational fluency is the ability to fluently switch among different representations of the same mathematical model (e.g., graph, table, equation, statement expressed in words) and to link these multiple representations in meaningful ways. According to Lesh and Zawojewski (2007), representational fluency requires that students not only learn to use and understand representational media, but also that they gain experience creating representations as part of problem-solving activities.

Over-Reliance on Real-Life Problems

While Sfard (2003) acknowledges a substantial body of research supporting the effectiveness of using real-life problem situations, she advises that mathematics instruction should not solely focus on real-life problem solving—because then students might get the message that "whatever has little practical importance does not have to be learned." She notes that more advanced mathematics typically taught at the high school level may not be "sufficiently necessary in everyone's life for usability to be the sole argument for learning it." Furthermore, teaching mathematics only based on real-world applications "is bound to lead to segmenting and impoverishing the subject matter," making it "unlikely that a complete, well-organized picture would eventually emerge" (Sfard, 2003).

Research-Based Recommendations for Mathematics Instruction

Recommendations based on the research sources reviewed in this section about incorporating problem solving in mathematics education include the following:

- Problem solving should provide a context for developing both conceptual understanding and procedural knowledge (Sfard, 2003).
- Instructional activities should often involve students in solving problems that require them to both think inventively and practice the skills they have already learned (Hiebert, 2003; Sfard, 2003). The problems should call upon students to draw upon the "system of concepts" that they already understand well (Sfard, 2003).
- Problem-solving activities should strike a balance in terms of the level of challenge—offering enough challenge to stimulate further development of conceptual understanding but not so much challenge to cause frustration. In this regard, individual differences among students should be taken into account (Sfard, 2003).
- Problem-solving activities should include representation of problems in mathematical models, such as diagrams, graphs, and equations (Lesh & Zawojewski, 2007; NCTM, 2000, 2014). Over time, problem solving should help students develop representational fluency among different representational forms (Lesh & Zawojewski, 2007).
- Mathematics instruction should encourage analysis of multiple methods of solving problems and comparison of methods (Hiebert, 2003).
- Educators should encourage thinking by requiring students to provide explanations of their method(s) of solving mathematics problems (Burns, 2015; Hiebert, 2003).

Research on Growth Mindset

A substantial body of research demonstrates that instruction that supports a growth mindset by encouraging persistence of effort is more effective than reinforcing the idea that intelligence and ability are fixed (Dweck, 2015).

Strong Research Support for a Growth Mindset

According to Dweck's (2015) research, students who believe their intelligence is a fixed construct are less motivated to apply themselves in school. They become more focused on appearing to be smart, and their beliefs make them see challenges and the need to exert effort as more of a threat rather than a way to improve their learning. By praising innate intelligence, teachers and parents reinforce students' views that their abilities are fixed, rather than malleable, and discourage students from striving to improve (Dweck, 2015).

Researchers have carried out multiple studies examining how students' theories about their own intelligence impact their learning (Blackwell et al., 2007; Robins & Pals, 2002; Stipek & Gralinski, 1996). A 1990 study found that students who embraced an incremental view of their own intelligence had higher grades in their first year of junior high school (Blackwell et al., 2007).

Studies by Dweck found that the students who were encouraged to succeed through increased effort and persistence after repeated failure came to see their failure as being due to insufficient effort rather than innate ability. By not viewing their intelligence as static, these students came to understand that if they persevered, they could succeed at tasks at which they had once failed (Dweck, 2015).

Two studies focused on 373 junior high school students over a two-year period to investigate how students' mindsets might impact their learning in mathematics. At the start of the first study, students' mindsets were assessed, and the researchers found that students with a growth mindset tended to experience an upward trajectory in grades, while students who believed their intelligence was fixed tended toward a flat trajectory. The second study designed an intervention to teach students how to develop a growth mindset. The results indicated that teaching students to have a growth mindset (referred to as an "incremental theory") resulted in improved motivation and a reversal in their downward trajectory of grades. Students in the control group were less motivated and continued on a downward trajectory in grades (Blackwell et al., 2007).

Blackwell et al. (2007) note that children begin developing fixed or growth mindsets in elementary school, but often problems do not arise until academic work becomes more challenging during early adolescence. Because elementary school is more failure-proof, students with a fixed mindset are less apt to experience negative consequences until they reach middle school.

Research-Based Recommendation for Mathematics Instruction

Dweck (2015) advises that teachers and instructional tools focus student praise on the steps they are taking to master learning material rather than on their innate intelligence:

> *Praise is very valuable . . . if it is carefully worded. Praise for the specific process a child used to accomplish something fosters motivation and confidence by focusing children on the actions that lead to success. Such process praise may involve commending effort, strategies, focus, persistence in the face of difficulty, and willingness to take on challenges* (Dweck, 2015).

How *i-Ready Personalized Instruction* for Mathematics Aligns to the Research

i-Ready Personalized Instruction for Mathematics addresses all of the critical domains specified in the NCTM and CCSS and follows key recommendations from these initiatives, founded in research and expert opinion:

- Promotion of meaning making in mathematics
- Integration of conceptual, factual, and procedural aspects of mathematics
- Providing problem-solving contexts for learning mathematics and representing problems in mathematical models
- Encouraging and providing support for persistent, productive struggle when attempting to solve mathematics problems

i-Ready Personalized Instruction for Mathematics Addresses All the Critical Mathematics Domains

In alignment with the NCTM and CCSS, *i-Ready Personalized Instruction* for Mathematics provides instruction and practice in these essential domains:

- Number and Operations
- Algebra and Algebraic Thinking
- Measurement and Data
- Geometry

The lesson sequence across Grades K–8 in each domain was informed by the *Progressions for the Common Core State Standards in Mathematics* (2011–2018).

i-Ready Personalized Instruction for Mathematics Promotes Meaning Making and Integrates Conceptual, Factual, and Procedural Learning in Problem-Solving Contexts

i-Ready Personalized Instruction for Mathematics lessons are designed to help students construct meaning around mathematics concepts and principles, basic mathematics facts, and procedures—and the structural relationships among these. The *i-Ready Diagnostic* assessment helps ensure that the lessons build on each student's entry knowledge. Lessons are designed around problem-solving challenges that call for a combination of invention and practice of previously learned procedures.

In *i-Ready Personalized Instruction* for Mathematics lessons, students are active builders of meaning.

i-Ready Personalized Instruction for Mathematics lessons include open and guided explorations that help students make connections while developing their understanding of new concepts and procedures, with scaffolded instructional feedback provided as needed.

For example, in a Grade 3 Measurement and Data lesson on picture graphs, students construct meaning through a series of activities that meet students in their ZPD, progressing from understanding data in a table and what it means (e.g., choosing a title, choosing a scale) to building a scaled picture graph.

A Grade 7 Geometry lesson presents students with the challenge of trying to construct a triangle from a given set of line segments. Students discover that not any three segments will make a triangle, that three specific segments can only result in one unique triangle, and that two segments can be parts of an infinite number of non-congruent triangles. Through multiple examples and explorations, students discover for themselves the rules of triangle construction.

i-Ready Personalized Instruction for Mathematics lessons guide students to see the structural relationships among mathematical concepts, facts, and procedures—and to see procedures as meaningful.

i-Ready Personalized Instruction for Mathematics lessons are designed to address and integrate conceptual understanding, factual knowledge, and procedural proficiency.

For example, students use virtual counters, models, and manipulatives to develop understanding of the concepts of counting, cardinality, and place value. Students then use these understandings to develop strategies and algorithms to add, subtract, multiply, and divide whole numbers. Methods that will generalize to become standard algorithms are developed, discussed, and explained initially using a visual model.

In early lessons on Measurement, students begin by comparing the lengths of two different concrete objects. For example, they measure the length of a pencil by lining up a series of quarters alongside it. From there, the concrete objects are replaced with more abstract ones—one-inch tiles. The process of measurement remains the same: students line up inch tiles next to the object they're trying to measure. Finally, students use the alignment of inch tiles to "build" a ruler so they develop a conceptual understanding of a ruler as a measurement tool.

In Number and Operations lessons focused on multiplying one- and two-digit numbers, students explore the distributive property, using the area model for multiplication as a way of breaking larger numbers into simpler problems. They break two-digit numbers by place value, creating partial products they add together for the final answer. By the time they are introduced to the standard algorithm for multiplication, their experience with partial products allows them to see the standard algorithm as a more efficient way of carrying out familiar operations. They come to understand the standard algorithm as a *meaningful* procedure.

i-Ready Personalized Instruction for Mathematics assesses students' entry knowledge and builds on it, while considering individual differences.

Students begin their *i-Ready* experience by taking the adaptive *i-Ready Diagnostic*, which assesses a broad range of mathematics skills across the four essential domains—Number and Operations, Algebra and Algebraic Thinking, Measurement and Data, and Geometry. Then *i-Ready* places each student in a personalized learning path through the online lessons in each domain. The customized learning path starts students with lessons in the domain(s) in which they placed the lowest, until they “catch up” and are working in all four domains at the same level.

The *i-Ready* online lessons are carefully sequenced to build on students’ prior knowledge. Individual *i-Ready* lessons respond to students’ actions and abilities. When students answer incorrectly during Personalized Instruction lessons, they are presented with feedback focusing on strategy and support. During *i-Ready* Practice lessons, the amount of practice with each item type is based on student performance with the prior items of that type. That way, each student receives the right amount of practice and can progress at an appropriate pace through the lessons.

Results from the *i-Ready Diagnostic* along with data from student performance during the online lessons are reported in ways that help teachers tailor their in-class instruction to the needs of their students. Reports indicate the levels at which students are working within each domain and where they’re struggling, and they provide grouping data for students working on the same challenges.

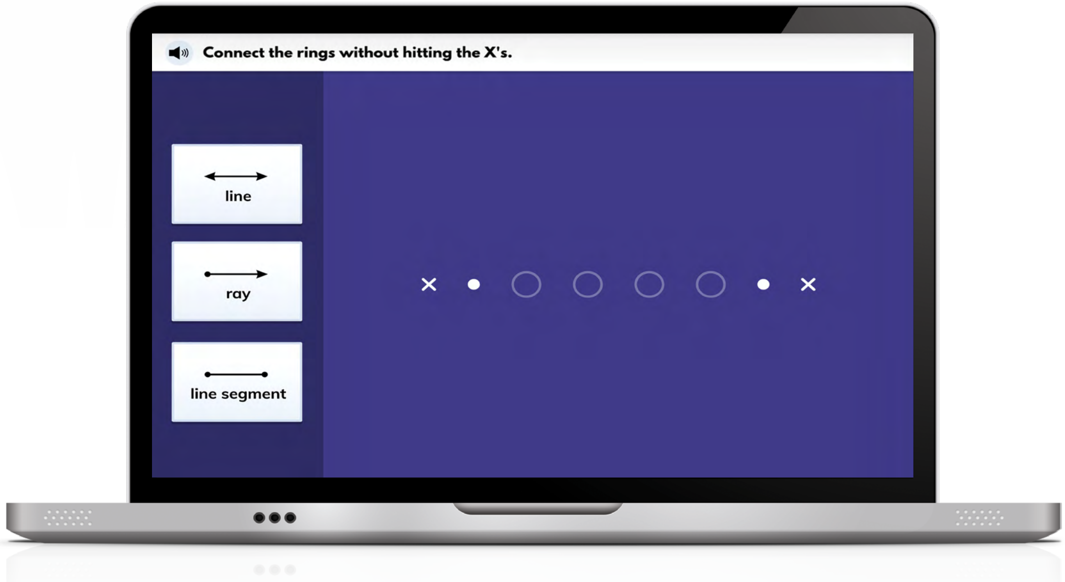
Problem solving that promotes conceptual understanding and application is central to *i-Ready Personalized Instruction* for Mathematics.

i-Ready lessons take a problem-solving approach to promote development of conceptual understanding. Where appropriate, Personalized Instruction lessons begin with an exploratory introduction, which presents a mathematical situation related to the topic of the lesson without an explicit explanation. This provides opportunities for students to apply their prior knowledge to a novel situation.

For example, when instruction focuses on the basic arithmetic operations, their meaning, and properties, students have opportunities to use virtual manipulatives, tape diagrams, arrays, and area models to solve a variety of problem types with unknowns in different places.

The idea of partial products is introduced by presenting an array model of multiplication and asking students if there is any way to partition the array to make the task easier. Students experiment by dividing the array to explore different configurations that might offer simpler numbers to work with.

A Geometry lesson on points, lines, and rays begins with a mini-game in which students decide which object (i.e., a line, line segment, or ray) to use to connect a series of dots while avoiding Xs in the field of play. Students gain experience with the properties of these objects prior to any explicit definition of the objects.



In *i-Ready Personalized Instruction* for Mathematics lessons, students are encouraged to explore multiple strategies for solving problems.

In *i-Ready Personalized Instruction* for Mathematics lessons, students are encouraged to try out multiple strategies for solving a problem.

For example, in Grade 2 lessons on adding and subtracting numbers up to 1,000, students use several strategies and/or combine strategies to find the solution. These strategies include breaking a number to add, making a 10 or power of 10, counting on to add, counting back to subtract, counting on to subtract (missing addend), and partial sums using place value. Students can also use an open number line and/or base-ten blocks.

Grade 7 lessons on ratios and percentages teach two different ways to solve multistep problems. For instance, to calculate a price p plus a tax of t%, students can calculate the:

- Tax as a percentage of the price: p x t% = tax. Then add the result to the price in order to find the total cost: p + tax = total cost.
- Total cost directly: p x (100 + t)% = total cost

i-Ready Personalized Instruction for Mathematics problem-solving activities provide appropriate challenge and significance to students.

The *i-Ready Diagnostic* assessment ensures that problems posed within *i-Ready Personalized Instruction* are at an appropriate level of challenge.

Open exploration activities that call for students to solve problems using their prior knowledge about the system of mathematical concepts are "significant" in the sense explained by Sfard (2003). In addition, *i-Ready* instructional designers strive to create mathematics problem scenarios that are meaningful and relevant to students. For example:

- A gym teacher divides her class into groups to play volleyball.
- A student checks out different types of books in the library.
- A school band marches in rows at a parade.
- Students in a class recycle plastic bottles.
- Students in a class volunteer to plant trees.
- Students run a car wash to raise money for their school.

A major focus of lesson development is to include culturally and linguistically responsive mathematics content so members of a diverse student population have opportunities to see themselves reflected in the problems they're working on. This is done through ethnic diversity in the names of characters in the problem scenarios, the foods they eat, the things they do, etc.

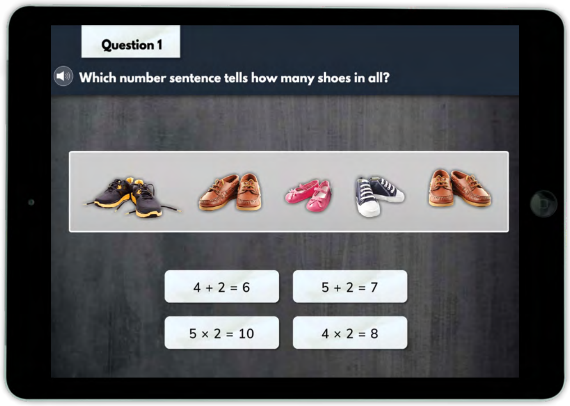
i-Ready Personalized Instruction **for Mathematics problem-solving activities involve representation of problems in mathematical models.**

i-Ready Personalized Instruction lessons present students with tools to use in modeling the problem at hand. The introduction of the tools helps students understand the relationship between concrete representations of problems and more abstract representations. In word-problem lessons, students have access to a toolbox and can choose from it which tool is most appropriate for modeling the current problem.

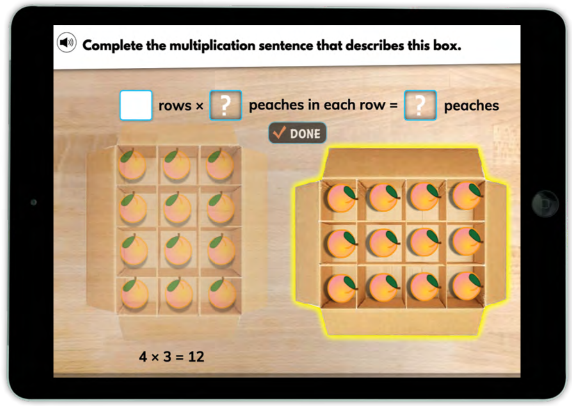
In some of the early lessons, students are presented with problems involving "how many" of certain objects. For example, a word-problem lesson focused on addition presents the problem: "Rico has 4 toy cars. He gets 3 more cars. How many cars does Rico have now?" The student is given a counters tool in which the *counters* look like two different types of cars. The student can model the situation with four cars of one type and three cars of the other. As the student's facility and familiarity with the counters tool progresses, counters that look like objects mentioned in the word problem are replaced with generic circle counters, and the student learns to use those in modeling any type of situation a problem presents.

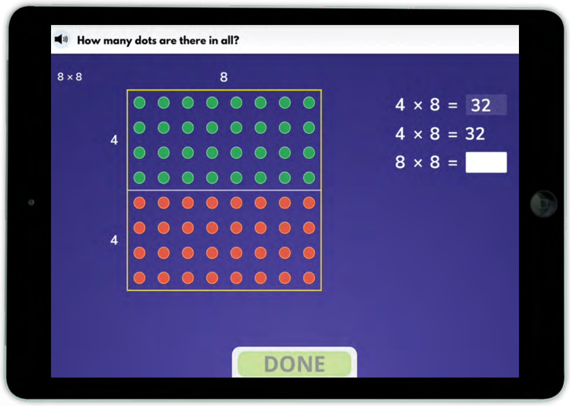
The progression of lessons focusing on multiplication starts with problem-specific objects in groups. After gaining experience with objects in groups, students start to see objects organized in arrays and focus on the dimensions of the arrays in order to represent the multiplication problem at hand. From problem-specific images in arrays, the lessons transition to the use of dots in arrays (i.e., an abstract representation). Finally, as numbers get increasingly large, multiplication is shown as the relationship of an array of squares to the area model, in which the dimensions become lengths rather than counts of rows and columns.

Equal Groups of Objects

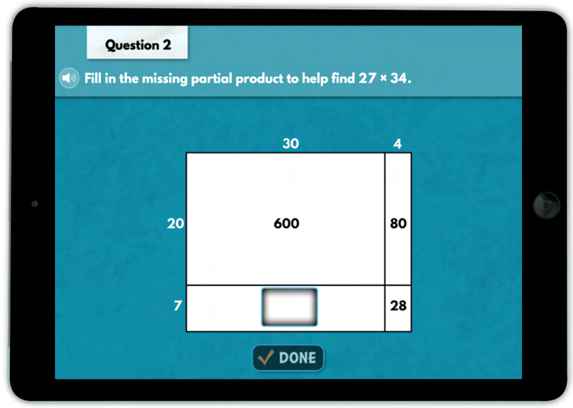


Arrays with Pictures of Concrete Objects





Arrays with Dots



Area Model

While working across the grade levels and mathematics domains, students are exposed to a broad collection of representations, including counters, array tools, bar models, measurement tools, inch tiles, area models, solid figures, tables, number lines, coordinate grids, base-ten blocks, place value charts, decimal grids, and fraction tiles.

If students struggle in their use of representational tools, the system guides them on a walkthrough of the process of using one of the tools to build a model of the current problem.

i-Ready Personalized Instruction for Mathematics lessons balance opportunities for problem solving with practice of previously learned skills.

i-Ready Personalized Instruction lessons include open and guided explorations that pose a problem to solve and use it as the context for developing students' understanding of new concepts or procedures, with scaffolded instructional feedback provided as needed. Independent Practice activities provide students with multiple opportunities to practice previously learned concepts, promote deeper understanding, and develop procedural fluency. Students continue to receive instructional feedback as necessary.

i-Ready Personalized Instruction for Mathematics provides opportunities for students to explain their problem-solving strategies.

The *i-Ready* team understands that classroom discussions provide the best opportunities for students to share their mathematics problem-solving strategies with their peers and teacher. To support this, *i-Ready* provides teachers with lesson plans in the Tools for Instruction—to be used independently of the online lessons—that encourage whole class or small group conversation about students' various strategies for solving mathematics problems.

i-Ready Personalized Instruction for Mathematics Promotes Perseverance and Effort

i-Ready Personalized Instruction for Mathematics instruction encourages students to persist in their efforts to understand mathematics concepts and solve problems, and they are praised for their efforts and perseverance.

i-Ready Personalized Instruction **for Mathematics encourages productive struggle.**

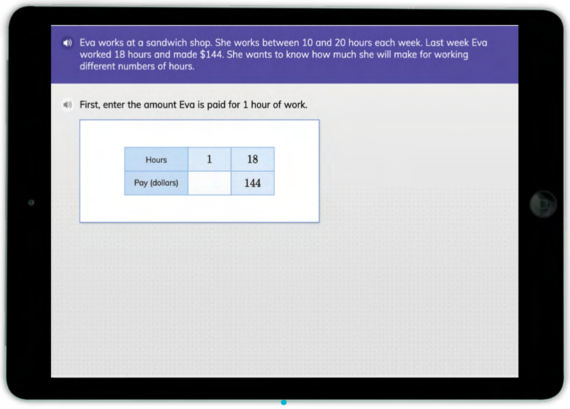
i-Ready Personalized Instruction for Mathematics activities offer students the opportunity to engage with challenging mathematics problems in a deep way, which involves the possibility of initial struggle. If students are not successful with a problem at first, scaffolded feedback supports students by suggesting strategies for successfully solving the problem and encouraging them to try again, rather than simply giving the answer. This helps students develop the sense that additional effort will lead to success.

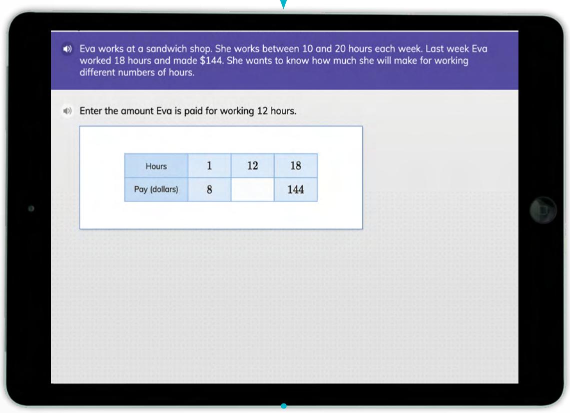
For example, in an early lesson on adding to subtract using a number line, the student is walked through the process of getting from 26 to 54 on the number line. The first step is, "How far is the jump from 26 to 30?" If the student answers incorrectly, the feedback is, "26 plus what number equals 30? Try again." This reframes the question, giving the student a potentially new approach to thinking about it.

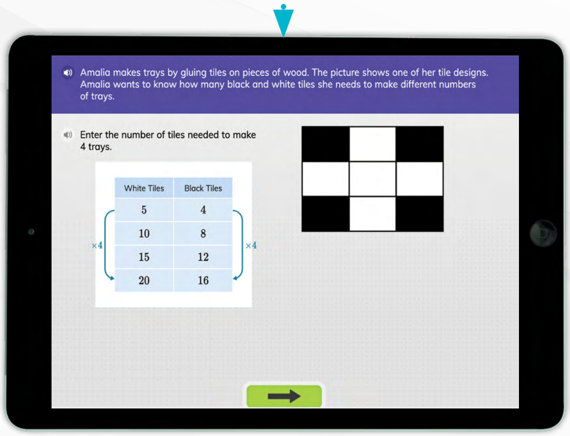
In a Grade 6 lesson introducing ratio tables, the first activity walks students through how to use the table to solve a ratio problem. The second activity guides students through different ways to solve problems using a ratio table. The third activity presents a problem and an open ratio table. Students can add columns and use the table however they want to solve the problem, with a walkthrough available if they don't know how to proceed.

Activity 1: How to Use a Ratio Table to Solve a Ratio Problem

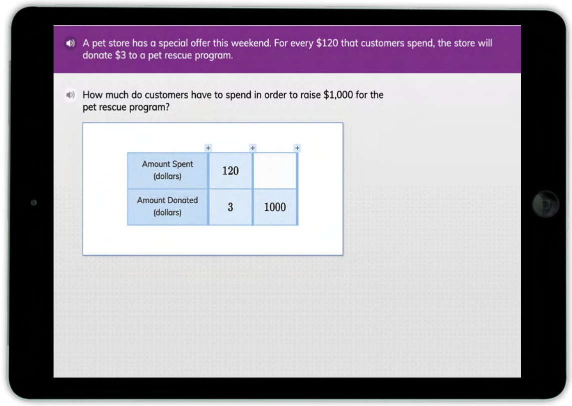
Activity 2: Different Ways to Solve Problems Using a Ratio Table



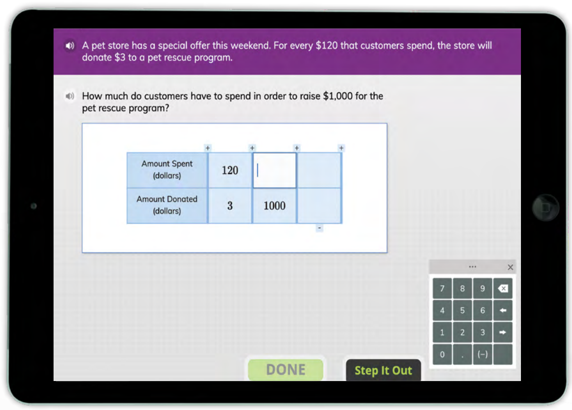




Activity 3: Given a problem and an open ratio table, the student can click on the plus signs to add columns and enter equivalent ratios.

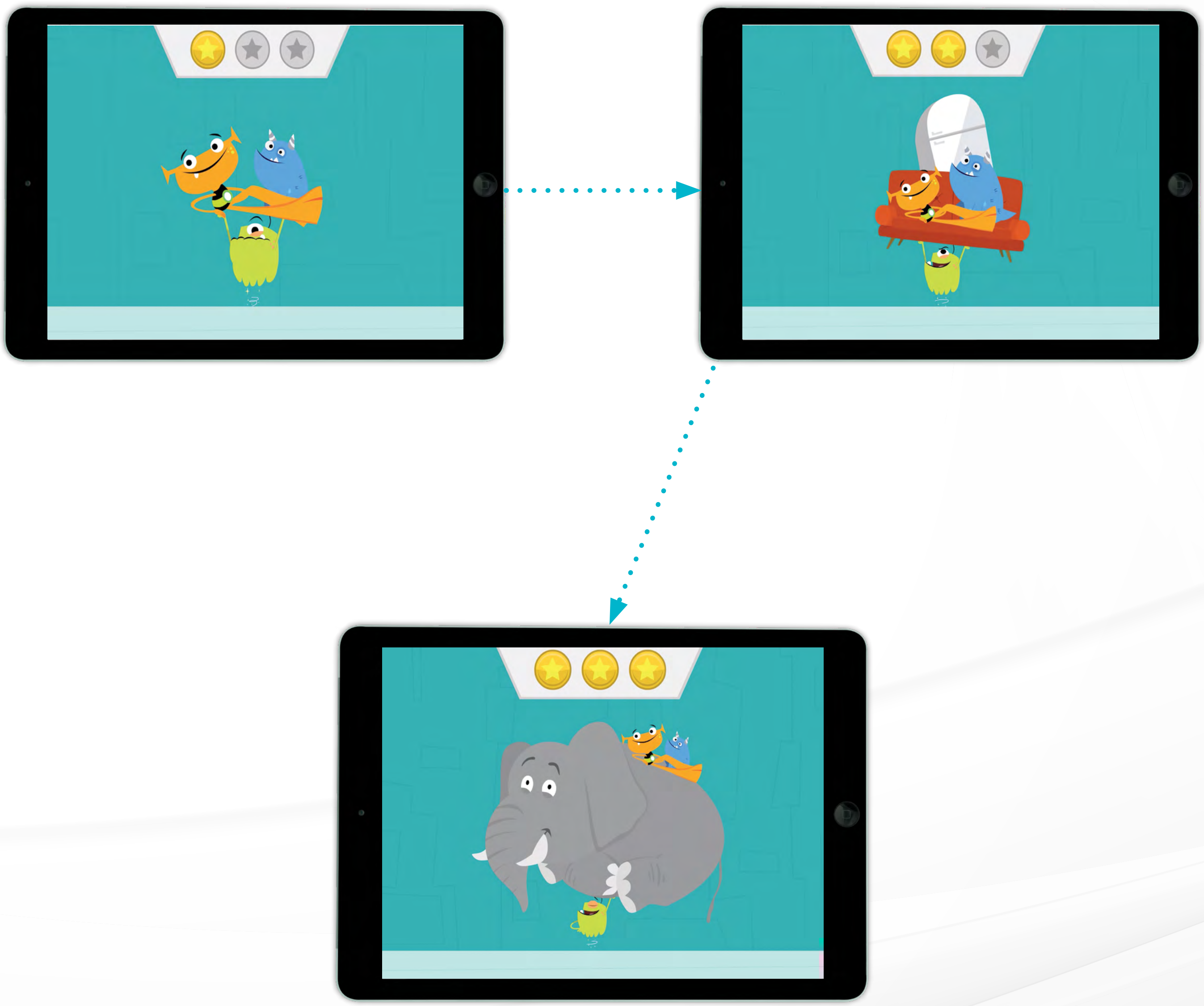


Activity 3: If incorrect on the first try, the student can continue trying on their own or click the Step It Out button to walk through a solution.



i-Ready Personalized Instruction **for Mathematics monitors students' progress and rewards effort.**

Progress monitoring screens for students in Grades K–2 reward completion of activities rather than initial correct responses. Students earn a star for each round of practice. Lessons are structured with scaffolded support to ensure each student is successful by the time they complete a lesson. Reward screens model the improvement in the onscreen character's ability as he puts in more effort over time:



The student dashboard allows students across the K–8 grade band to track the amount of time they've spent on the system and the number of lessons they've completed. This serves as a record of their effort over time.

References

ACT (2019). Condition of college and career readiness 2019. Retrieved from https://www.act.org/content/dam/act/unsecured/documents/National-CCCR-2019.pdf.

Baroody, A. J., & Ginsburg, H. P. (1986). The relationship between initial meaningful and mechanical knowledge of arithmetic. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 75–112). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Blackwell, L. S., Trzesniewski, K. H., & Dweck, C. S. (2007). Implicit theories of intelligence predict achievement across an adolescent transition: A longitudinal study and an intervention. *Child Development*, *78*(1), 246–263. Retrieved from https://www.mtoliveboe.org/cmsAdmin/uploads/blackwell-theories-of-intelligence-child-dev-2007.pdf. doi:10.1111/j.1467-8624.2007.00995.x.

Briars, D. (2014). Back to school: The time to engage parents and families. Retrieved from https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Diane-Briars/Back-to-School_-The-Time-to-Engage-Parents-and-Families/.

Burns, M. (1998). *Math: Facing an American phobia*. Sausalito, CA: Mathematics Solutions.

Burns, M. (2015). *About teaching mathematics* (4th ed.). Sausalito, CA: Mathematics Solutions.

Common Core Standards Writing Team. (2011–2018). *Progressions for the Common Core State Standards in mathematics*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Common Core State Standards (CCSS) Initiative (2019). About the standards. Retrieved from http://www.corestandards.org/about-the-standards/.

Curriculum Associates (2017). *i-Ready efficacy: Research on i-Ready program impact. Research support for i-Ready and ESSA*. Retrieved from https://www.casamples.com/downloads/iready-essa-brochure-2017.pdf.

Dweck, C. S. (2015). The secret to raising smart kids. *Scientific American Mind*, *Special Editions, 23*(5s), 76–83. Retrieved from https://www.scientificamerican.com/article/the-secret-to-raising-smart-kids1/. doi:10.1038/scientificamericangenius0115-76.

Hiebert, J. (2003). What research says about the NCTM Standards. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 5–23). Reston, VA: The National Council of Teachers of Mathematics.

Kilpatrick, J., Martin, W. G., & Schifter, D. (Eds.) (2003). *A research companion to principles and standards for school mathematics*. Reston, VA: The National Council of Teachers of Mathematics.

Lesh, R., & Zawojewski, J. (2007). Problem solving and modeling. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Assessment of Educational Progress (NAEP) (2019). The nation's report card: 2019 NAEP mathematics assessment. Retrieved from https://www.nationsreportcard.gov/highlights/mathematics/2019/.

National Council of Teachers of Mathematics (NCTM) (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

NCTM (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author.

National Governors Association Center for Best Practices (NGACBP) & Council of Chief State School Officers (CCSSO) (2010). *Common Core State Standards for mathematics*. Washington, DC: NGACBP, CCSSO. Retrieved from http://www.corestandards.org/wp-content/uploads/Math_Standards1.pdf.

National Research Council (NRC) (1989). *Everybody counts: A report to the nation on the future of mathematics education*. Washington, DC: National Academies Press. doi:10.17226/1199.

NRC (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academies Press. doi:10.17226/9822.

Sfard, A. (2003). Balancing the unbalanceable: The NCTM Standards in light of theories of learning mathematics. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 353–392). Reston, VA: The National Council of Teachers of Mathematics.

Curriculum Associates