Modeling and Optimization for Net Zero: Aligning with EU Climate Policy and Regulatory Frameworks

Learning from the SPaC experience

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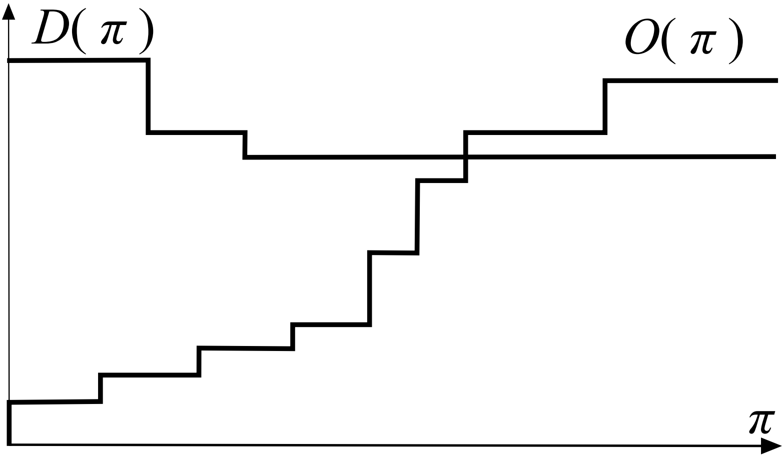
Outline

The Pay-as-Clear market clearing mechanism

- Market = sellers + buyers of a fungible divisible commodity (energy)
  - set S of sell offers $\langle\, sp_j \,,\, sq_j \,\rangle$: will sell ($\leq$) sq_j for a price $\geq sp_j$
  - set B of purchase bids $\langle\, bp_i \,,\, bq_i \,\rangle$: will buy ($\leq$) bq_i for a price $\leq bp_i$
- Nondecreasing offer curve (not function) $O(\pi) = \sum_{j \,:\, sp_j \,\geq\, \pi} sq_j$
- Nonincreasing demand curve (not function) $D(\pi) = \sum_{j \,:\, bp_j \,\leq\, \pi} bq_j$
- Clearing price $\pi^* =$ "where $O(\pi)$ and $D(\pi)$ meet" $\Longrightarrow$ total amount q^* (of energy) exchanged over the market
- Forget about market failures and degeneracy ...
- But why Pay-as-Clear?

The Pay-as-Clear Model, graphically



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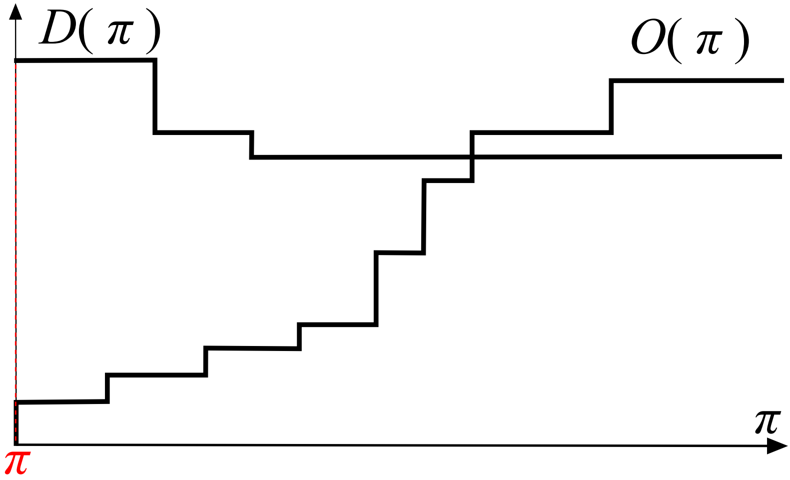

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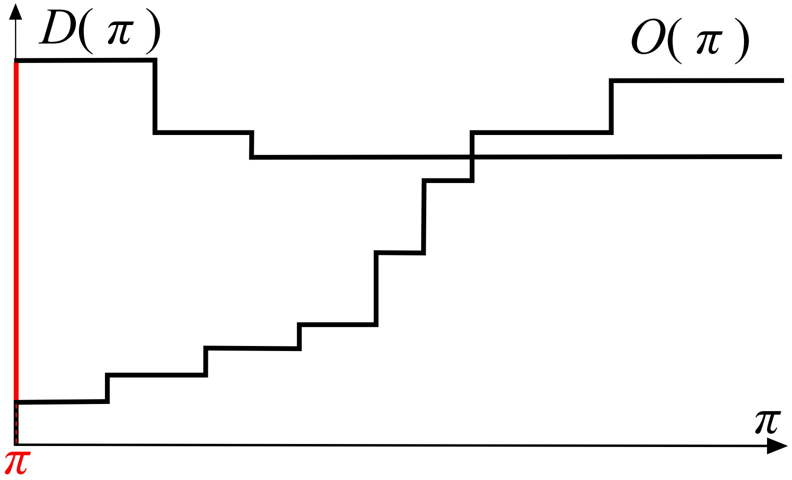

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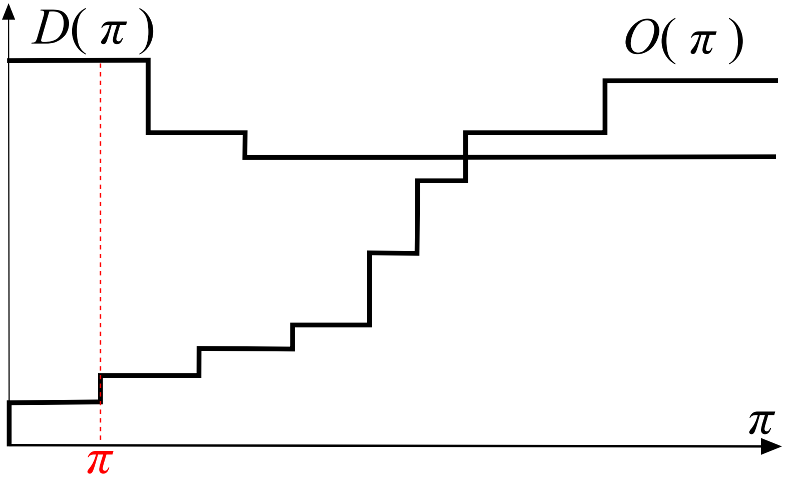

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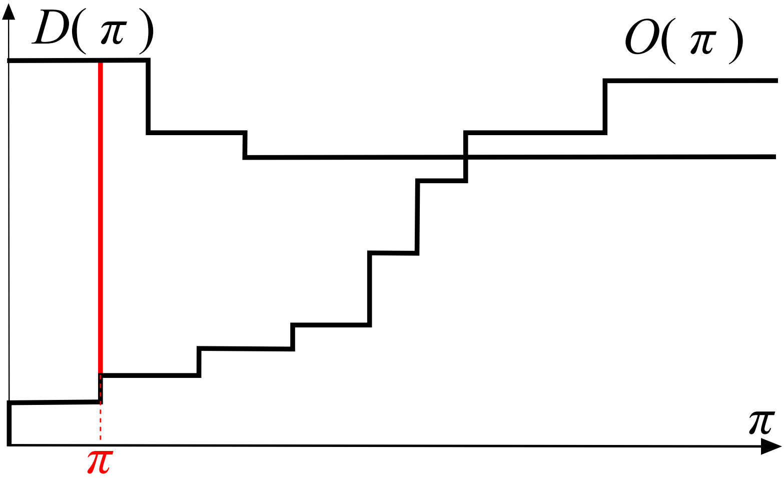

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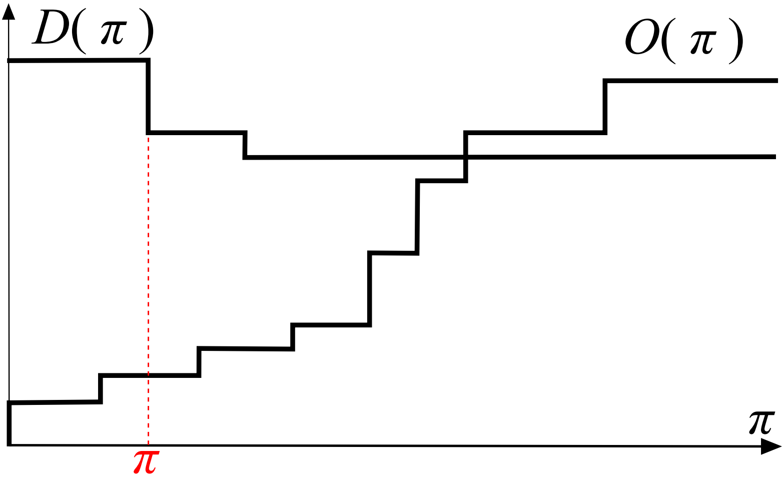

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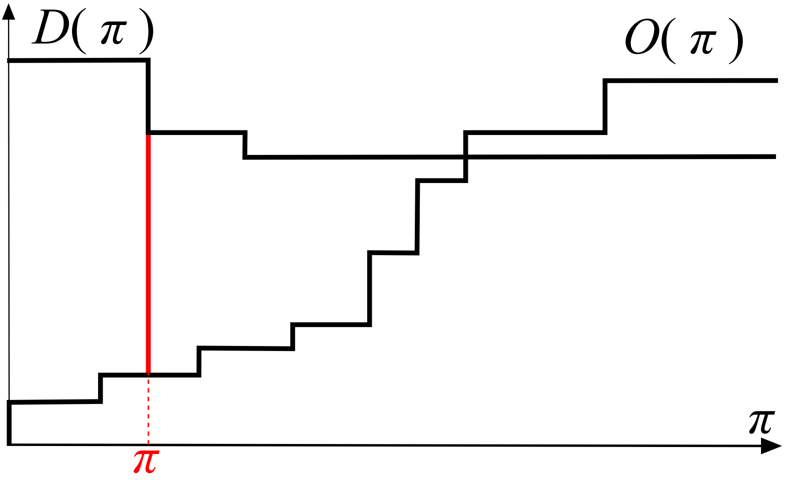

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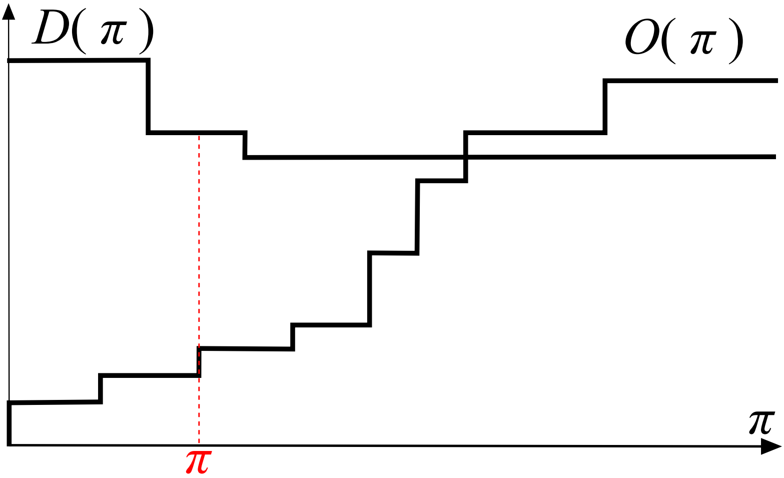

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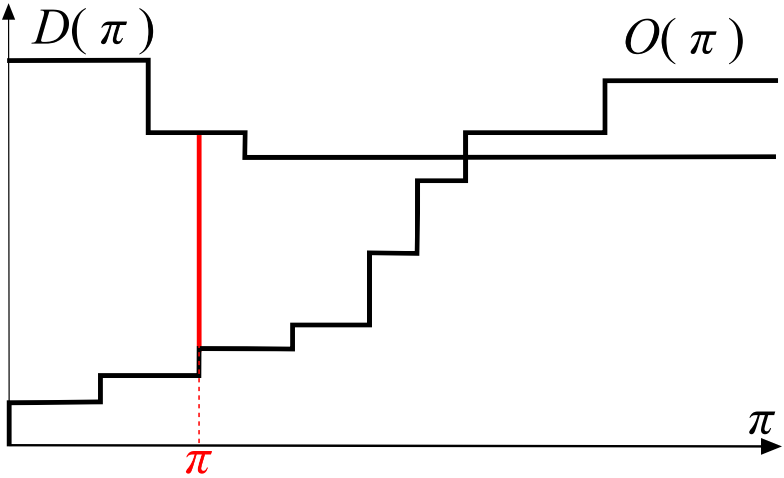

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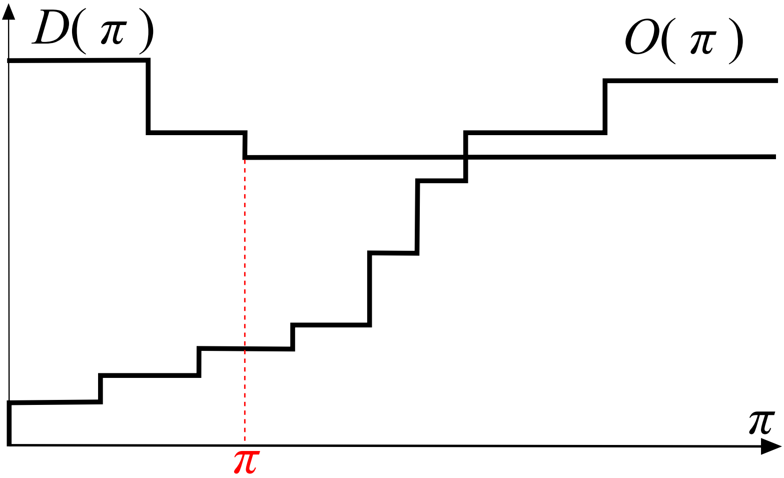

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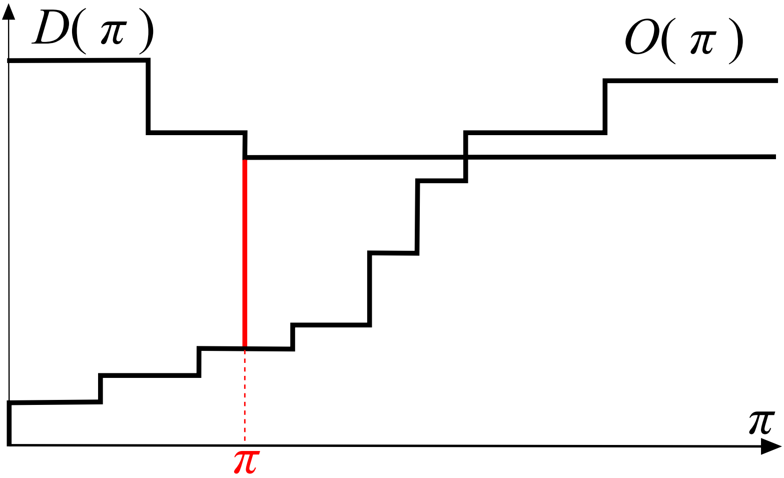

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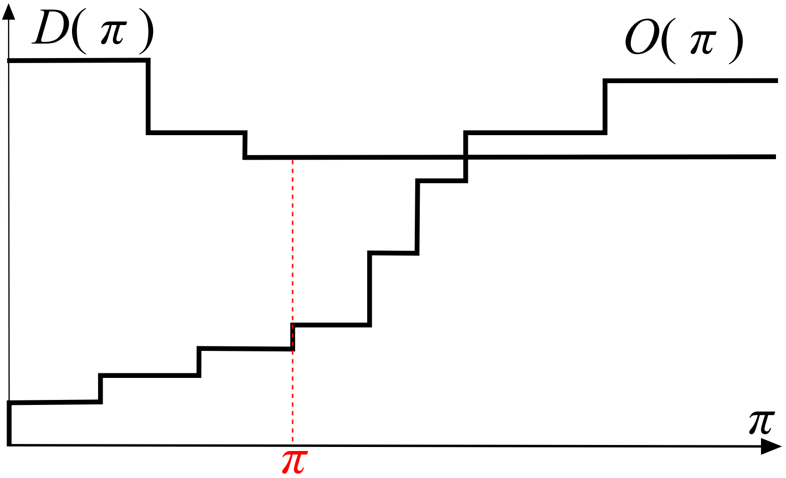

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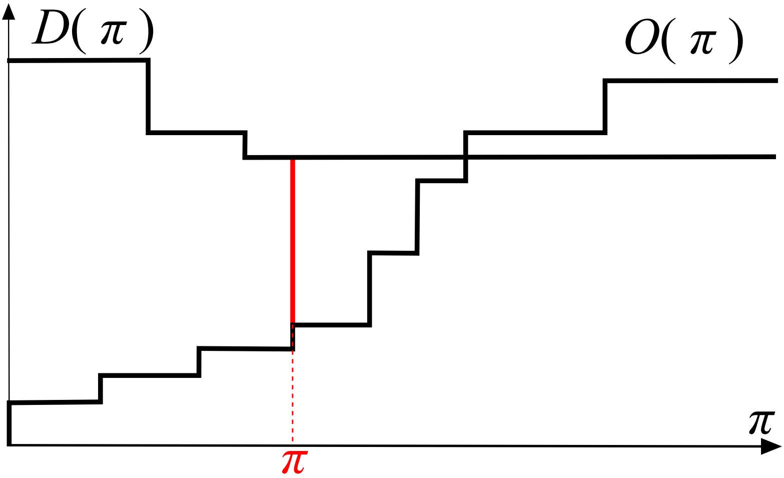

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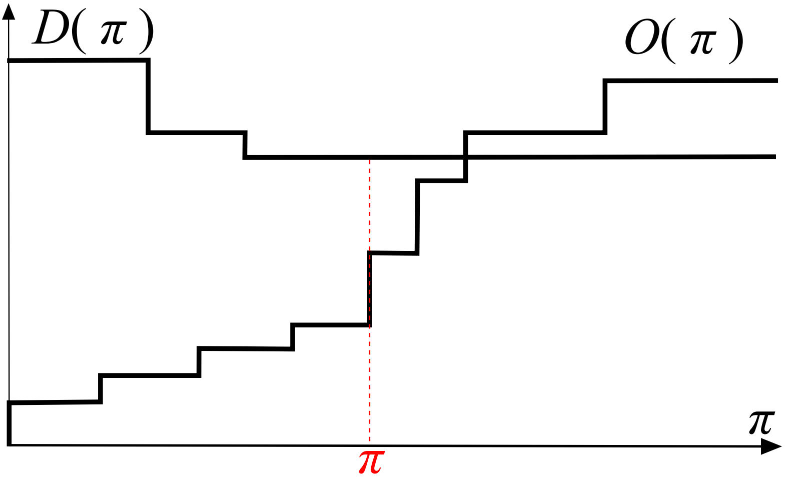

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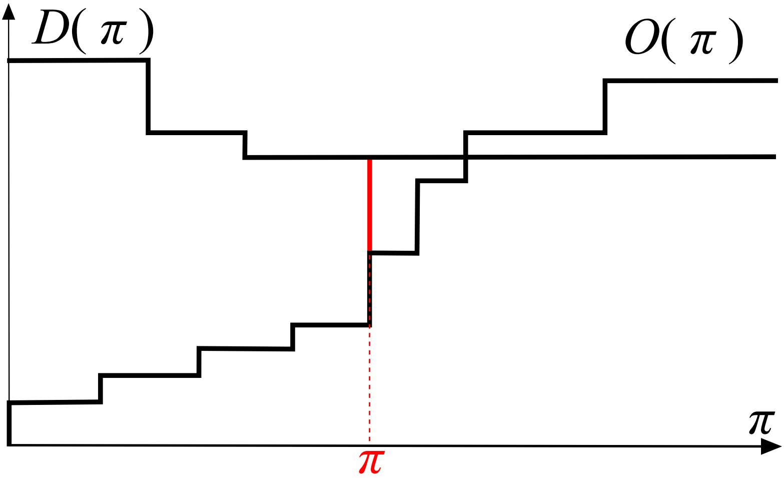

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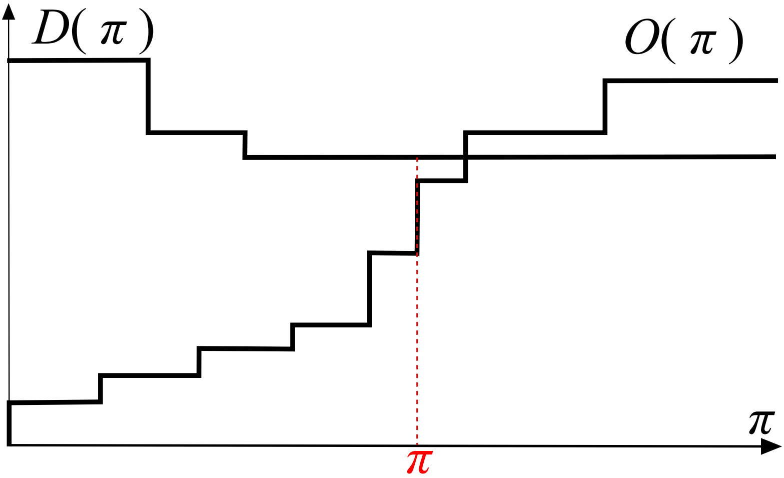

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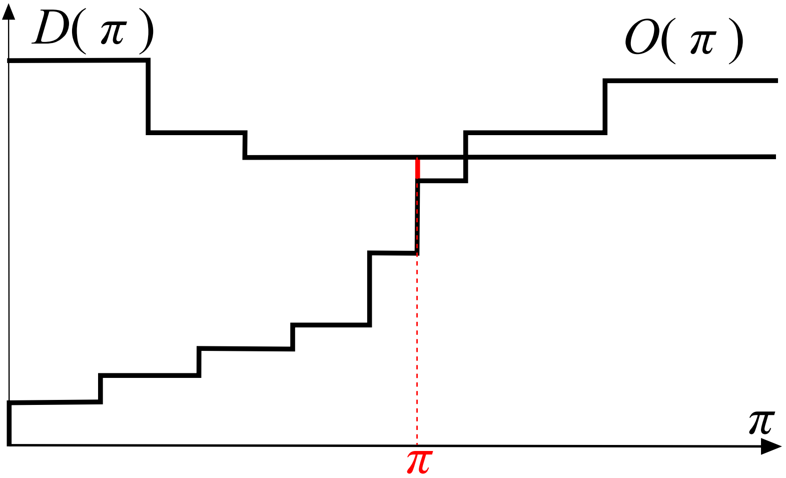

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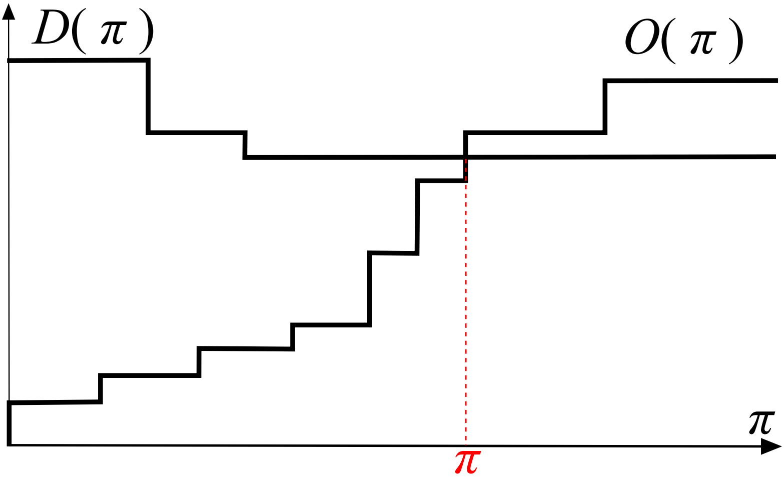

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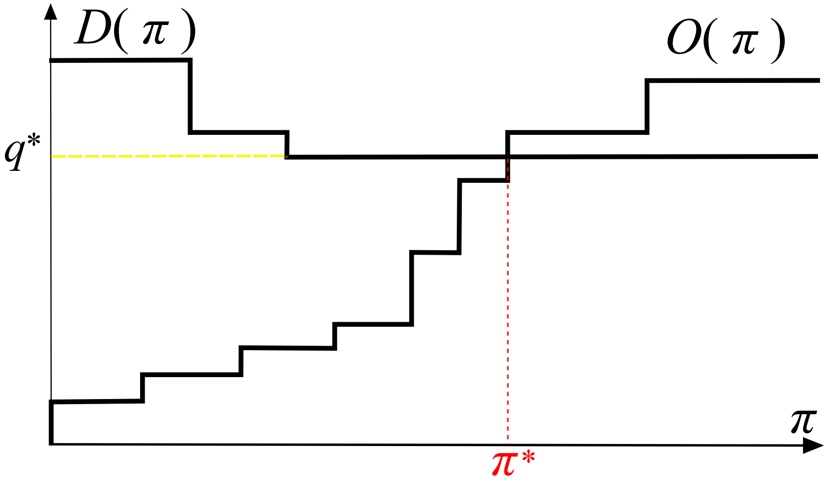

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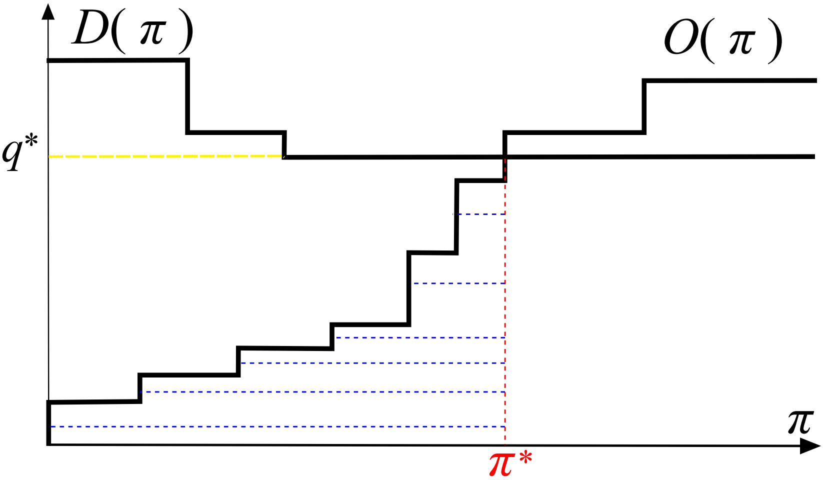
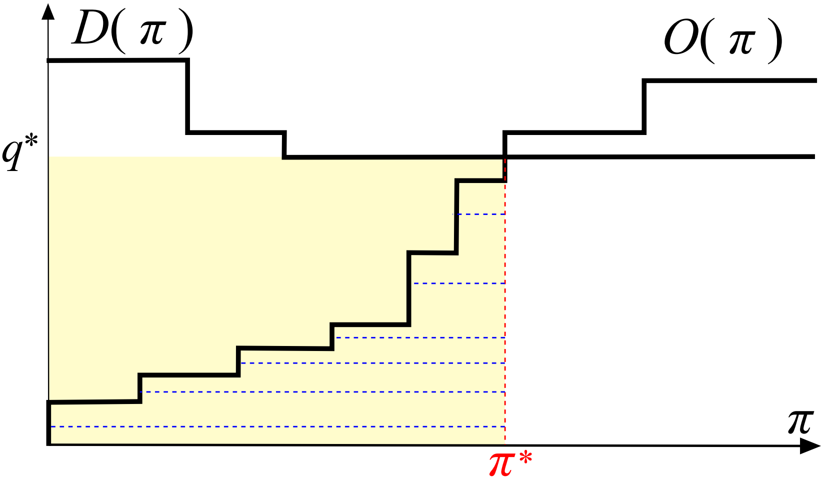


- Everyone paid at the clearing price π^*

The Pay-as-Clear Model, graphically



- Everyone paid at the clearing price π^* $\Longrightarrow$ total system cost $= \pi^* q^*$

Everyone loves it because it's an LP

- Let's simplify: fixed demand $\equiv$ only sell offers ($\approx$ true in electricity)
- Primal / dual market clearing problems:

$$\min \sum_{j \in S} sp_j s_j \quad (1)$$
$$0 \leq s_j \leq sq_j \quad j \in S \quad (2)$$
$$\sum_{j \in S} s_j = d \quad (3)$$

$$\max \sum_{j \in S} sq_j \eta_j + \pi d \quad (4)$$
$$\eta_j + \pi \leq sp_j \ , \ \eta_j \leq 0 \quad j \in S \quad (5)$$

- Primal feasibility + dual feasibility + complementary slackness

$$\eta_j (s_j - sq_j) = 0 \qquad j \in S \qquad (6)$$
$$(sp_j - \eta_j - \pi) s_j = 0 \qquad j \in S \qquad (7)$$

$\Longrightarrow$ optimal π^* the market clearing price

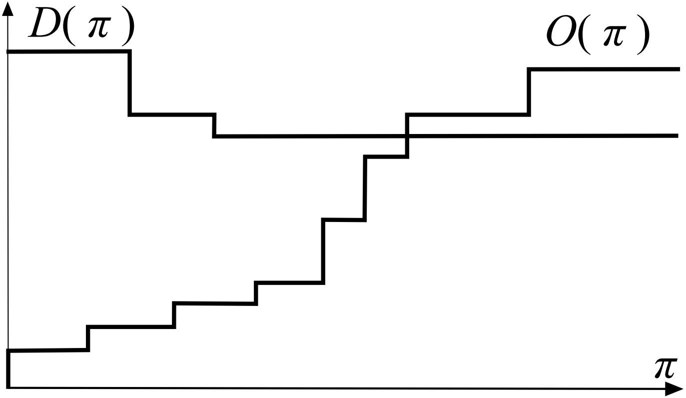
- Easy to see with just a bit of logic

It has many nice properties

- Day-Ahead Market solved every day for every hour of the next day (plus primary/secondary reserve markets, ancillary services, ...)
- Long-term average gives long-term price signal: how much is worth investing in new generation (5+y to build, 10+y amortization, ...)
- Hourly price gives short-term price signal: how much energy is worth in this specific hour, crucial for Unit Commitment (peak shaving ...)
- Pay-as-bid (apparently) not as good (don't ask ...)
- Can resist complications: variable demand, (DC) network constraints, strange market constructs (unique national price, complex bids, ...) because it's an LP or MPCC $\equiv \mathcal{NP}$-hard, but we are happy with that
- Everyone's happy then, so what's the problem?

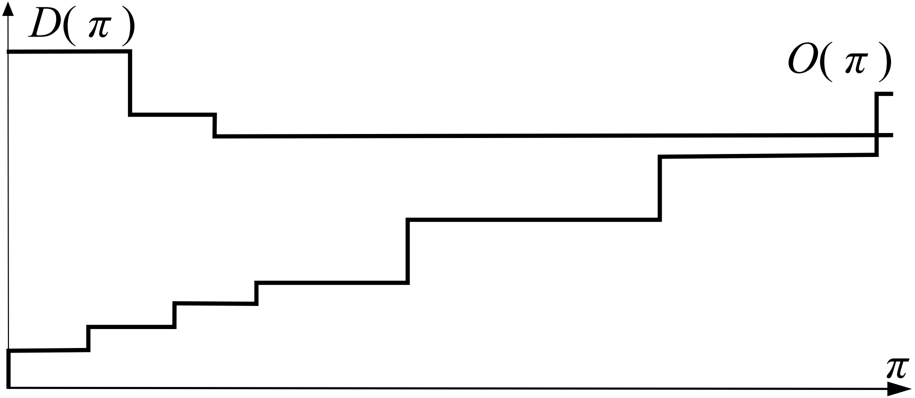
Outline

The Problem – graphically



- W.r.t. "normal" times

The Problem – graphically



- W.r.t. "normal" times gas prices shot up

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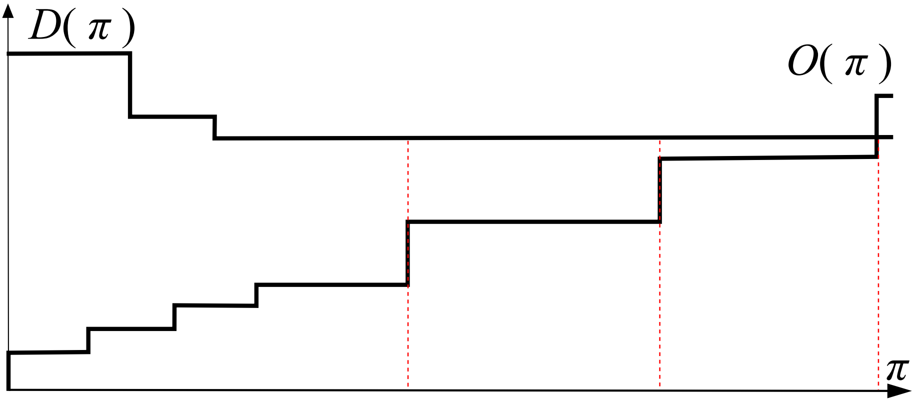

- W.r.t. "normal" times gas prices shot up $\Longrightarrow$ gas-fired units increased sp_j

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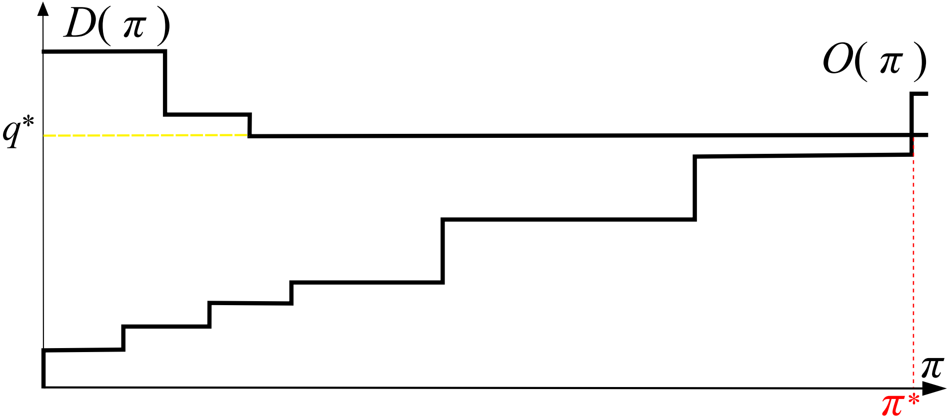

- W.r.t. “normal” times gas prices shot up $\Longrightarrow$ gas-fired units increased sp_j
- π^* shot up,

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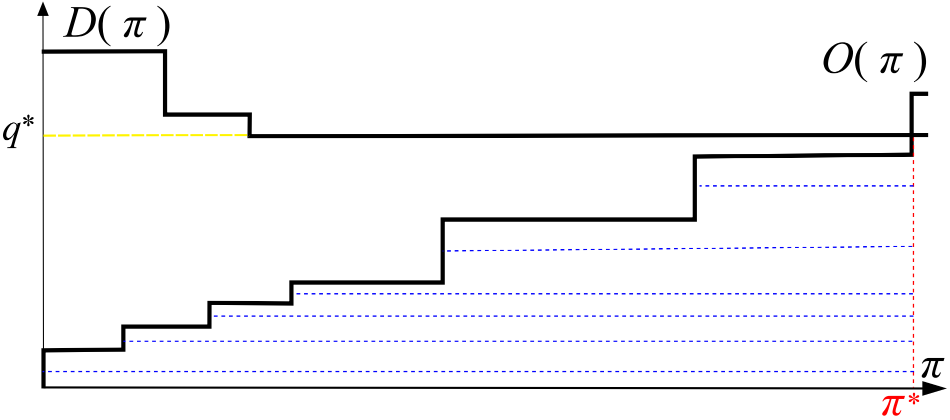

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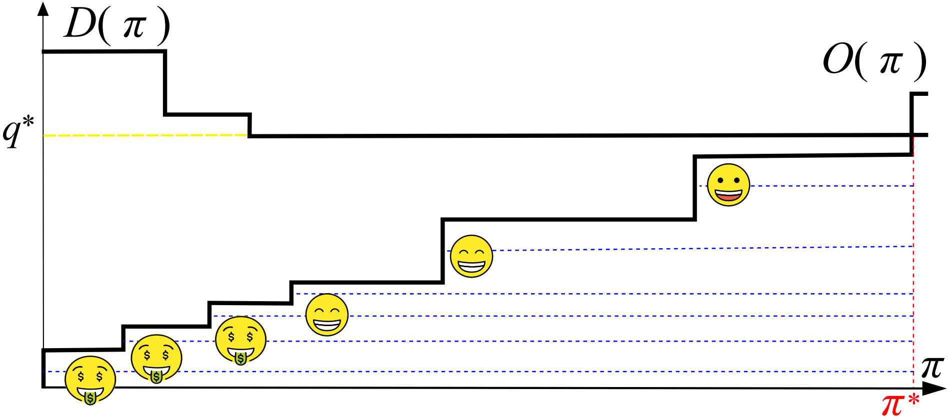

- W.r.t. "normal" times gas prices shot up $\Longrightarrow$ gas-fired units increased sp_j
- π^* shot up, producers corked spumante,

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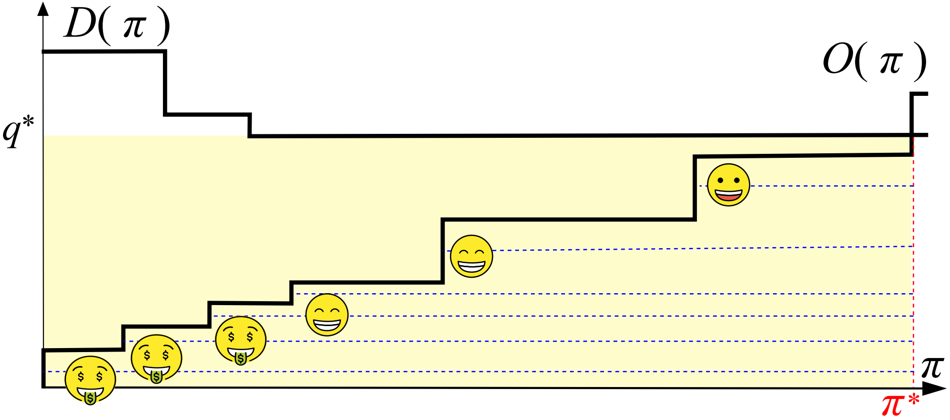

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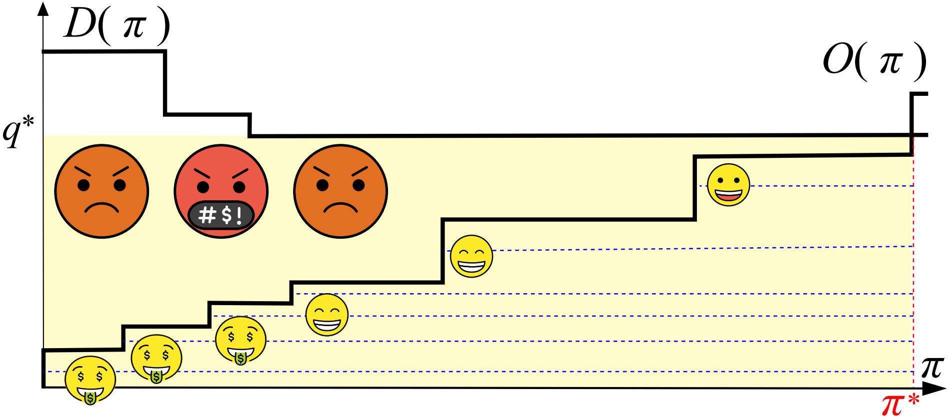

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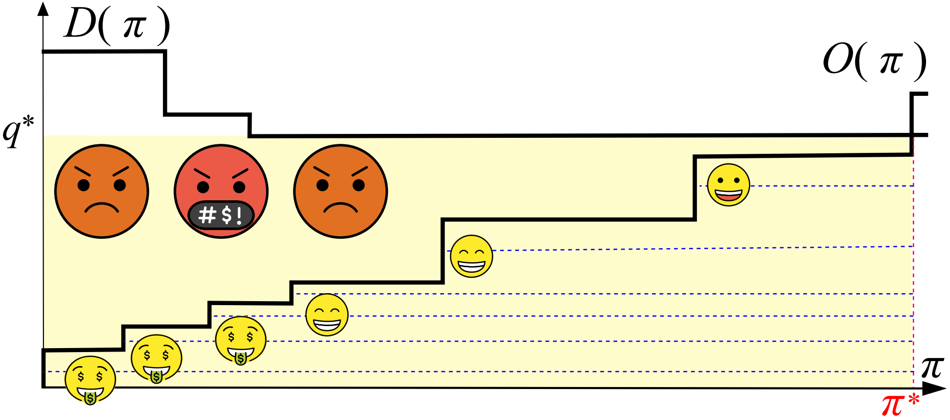

- W.r.t. "normal" times gas prices shot up $\Longrightarrow$ gas-fired units increased sp_j
- π^* shot up, producers corked spumante, consumers went down in flames
- The real energy cost had increased way less than the clearing price $\Longrightarrow$ not a functioning market, doing net-0 by creating an industrial wasteland

What one would want

- Partition $S = S^r \cup S^g$: S^r = reserved (renewables) market,
  S^g = general (gas-fired) market
- Have producers in each market only slog it out among themselves $\Longrightarrow$ different prices for the same commodity, reflecting fundamentally different cost structure of sets of producers
- Both markets must satisfy the same demand
- Economists were sharpening forks and lighting up pyres, but that was not what was bothering me
- How can you have two markets be separate, and then "magically" agree on the demand each will satisfy?
- I thought it was not possible ...

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- I thought it was not possible ... but Fabrizio knew better

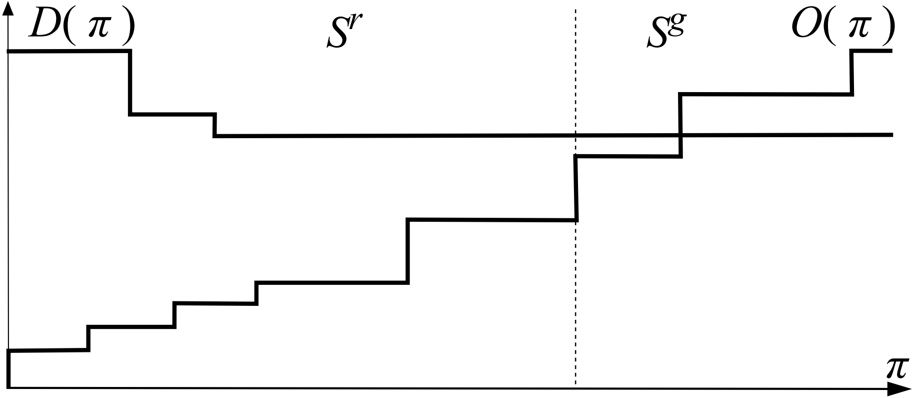
Outline

Segmented-Pay-as-Clear, version I: bilevel program

$$
\begin{aligned}
\min_{d^r, d^g} \quad & \pi^r d^r + \pi^g d^g \\
& d^r + d^g = d \;,\quad d^r \geq 0 \;,\quad d^g \geq 0 \\
& \pi^r \in \left\{ \begin{array}{ll} \arg\max_{\pi^r, \eta} & \sum_{j \in S^r} sq_j \eta_j \;+\; \pi^r d^r \\ & \eta_j + \pi^r \leq sp_j \,,\; \eta_j \leq 0 \quad j \in S^r \end{array} \right. \\
& \pi^g \in \left\{ \begin{array}{ll} \arg\max_{\pi^g, \eta} & \sum_{j \in S^g} sq_j \eta_j \;+\; \pi^g d^g \\ & \eta_j + \pi^g \leq sp_j \,,\; \eta_j \leq 0 \quad j \in S^g \end{array} \right.
\end{aligned}
$$

- The two markets compete among them for the demand
- Producers in each market compete among them as usual
  but not directly with producers in the other market
- The objective is bilinear (nonconvex), but bilevels are hard anyway:
  throw it to `Gurobi` via `BilevelJump`, it'll eat it
- Cannot do worse than PaC (will be obvious shortly)

Segmented-Pay-as-Clear, graphically



- Sell offers are partitioned $S^r \cup S^g$

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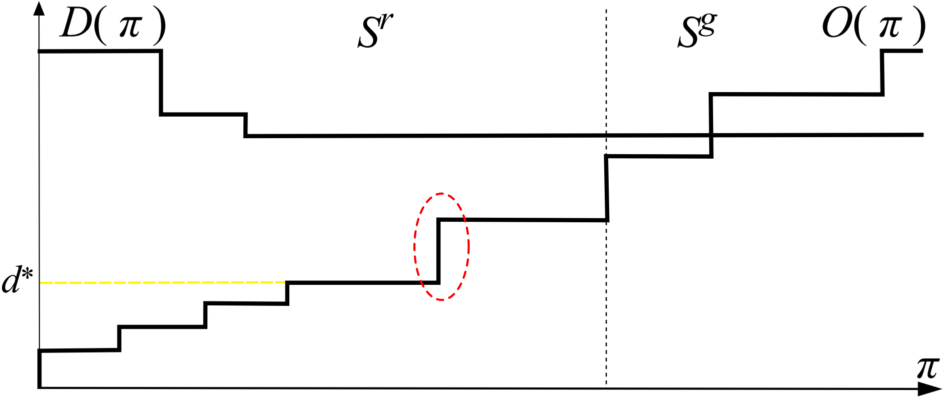

- Sell offers are partitioned $S^r \cup S^g$
- Playing with demand, "costly" bids $\in S^r$

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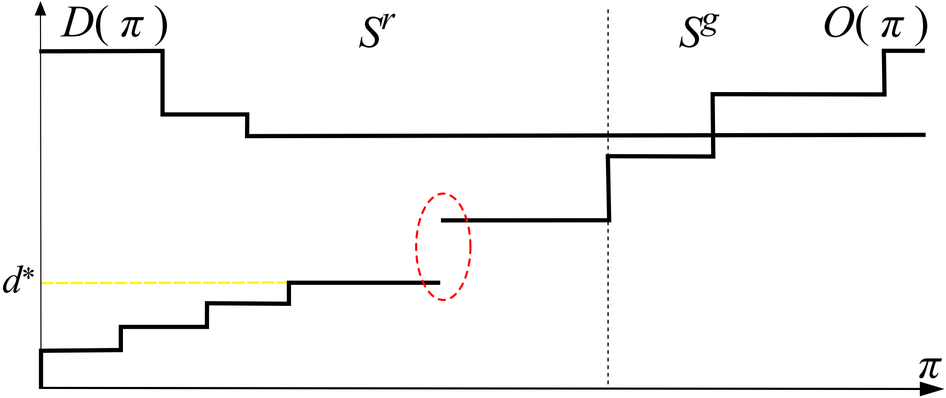

- Sell offers are partitioned $S^r \cup S^g$
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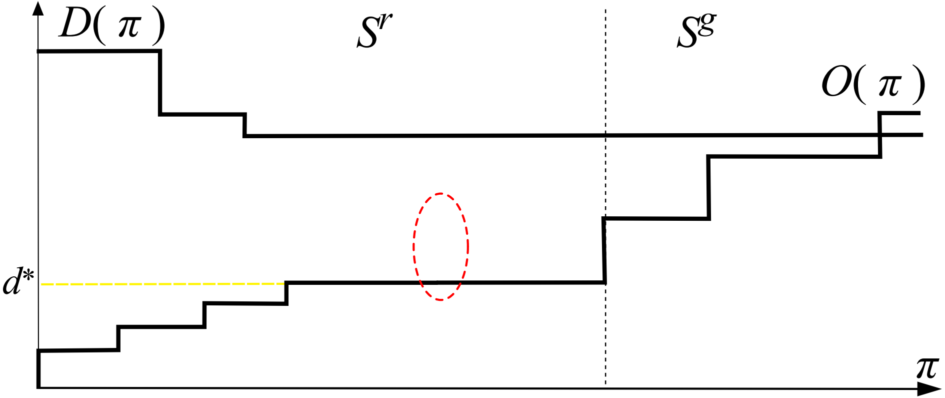

- Sell offers are partitioned $S^r \cup S^g$
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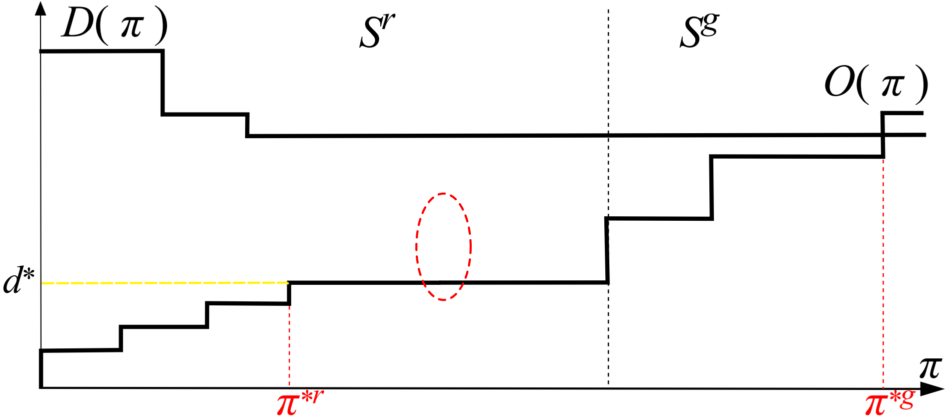

- Sell offers are partitioned $S^r \cup S^g$
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- Each market is separately cleared,

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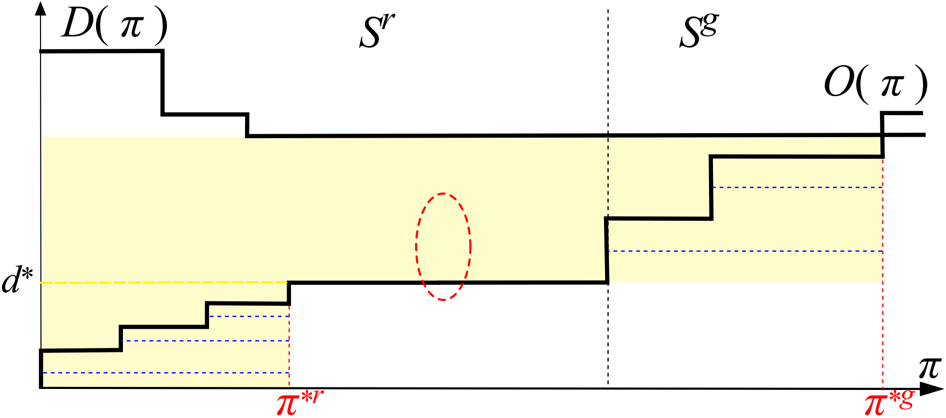

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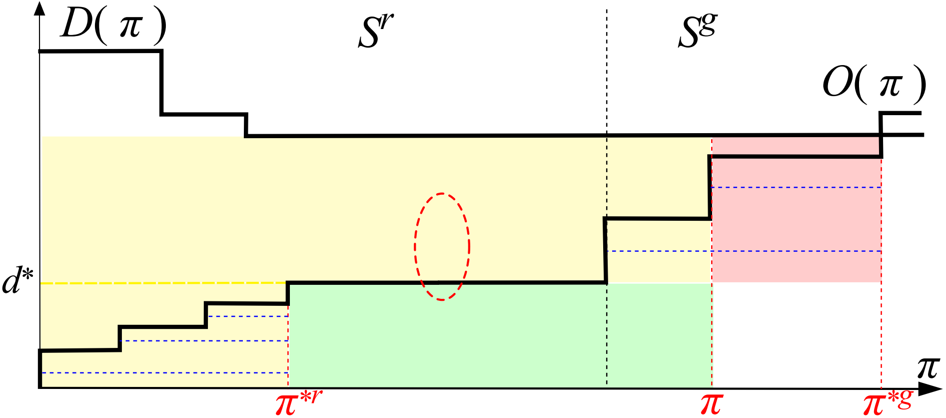


- Sell offers are partitioned $S^r \cup S^g$
- Playing with demand, "costly" bids $\in S^r$ can be "killed", changing $O(\pi)$
- Each market is separately cleared, total price is the sum
- Although $\pi^{*,g} > \pi^*$, $\pi^{*,s} < \pi^*$ and the sum may be less

Segmented-Pay-as-Clear, version II: MPCC

$$\min \pi^r d^r + \pi^g d^g \tag{8}$$

$$d^r + d^g = d \quad , \quad d^r \geq 0 \quad , \quad d^g \geq 0 \tag{9}$$

$$0 \leq s_j \leq sq_j \qquad j \in S \tag{2}$$

$$\textstyle\sum_{j \in S^r} s_j = d^r \tag{10}$$

$$\eta_j + \pi^r \leq sp_j \quad , \quad \eta_j \leq 0 \qquad j \in S^r \tag{11}$$

$$\textstyle\sum_{j \in S^g} s_j = d^g \tag{12}$$

$$\eta_j + \pi^g \leq sp_j \quad , \quad \eta_j \leq 0 \qquad j \in S^g \tag{13}$$

$$\eta_j(s_j - sq_j) = 0 \qquad j \in S \tag{14}$$

$$(sp_j - \eta_j - \pi^r)s_j = 0 \qquad j \in S^r \tag{15}$$

$$(sp_j - \eta_j - \pi^g)s_j = 0 \qquad j \in S^g \tag{16}$$

- Bilinear objective (8) and complementarity constraints (14)–(16)

Segmented-Pay-as-Clear, version II: MPCC

$$\min \pi^r d^r + \pi^g d^g = \sum_{j \in S}(sp_j s_j - \eta_j sq_j) \tag{8}$$

$$d^r + d^g = d \quad , \quad d^r \geq 0 \quad , \quad d^g \geq 0 \tag{9}$$

$$0 \leq s_j \leq sq_j \qquad j \in S \tag{2}$$

$$\sum_{j \in S^r} s_j = d^r \tag{10}$$

$$\eta_j + \pi^r \leq sp_j \quad , \quad \eta_j \leq 0 \qquad j \in S^r \tag{11}$$

$$\sum_{j \in S^g} s_j = d^g \tag{12}$$

$$\eta_j + \pi^g \leq sp_j \quad , \quad \eta_j \leq 0 \qquad j \in S^g \tag{13}$$

$$\eta_j(s_j - sq_j) = 0 \qquad j \in S \tag{14}$$

$$(sp_j - \eta_j - \pi^r)s_j = 0 \qquad j \in S^r \tag{15}$$

$$(sp_j - \eta_j - \pi^g)s_j = 0 \qquad j \in S^g \tag{16}$$

- Bilinear objective (8) and complementarity constraints (14)–(16)
- But one bilinearity can kill the other (thanks Medhi Madani)

Segmented Prices-as-Clear, the Final Reformulation

- Only one market, but with a limit on the energy from S^r:

$$\min \ \sum_{j \in S} sp_j s_j \qquad (1)$$
$$0 \leq s_j \leq sq_j \quad j \in S \qquad (2)$$
$$\sum_{j \in S^r} s_j \leq d^r \qquad (17)$$
$$\sum_{j \in S} s_j = d \qquad (3)$$

$$\max \ \sum_{j \in S} sq_j \eta_j \ + \ \pi d \ + \ \pi^r d^r \qquad (18)$$
$$\eta_j + \pi \leq sp_j \qquad j \in S^g \qquad (19)$$
$$\eta_j + \pi + \pi^r \leq sp_j \qquad j \in S^r \qquad (20)$$
$$\eta_j \leq 0 \qquad j \in S \qquad (21)$$
$$\pi^r \leq 0 \qquad (22)$$

- g-market clears at π, r-market clears at $\pi + \pi^r < \pi$ (cf. (22)) $\Longrightarrow$ cannot be worse than PaC, equal if d^r "too large" $\Longrightarrow$ $\pi^r = 0$ $\Longrightarrow$
$$(\pi + \pi^r) \sum_{j \in S^r} s_j + \pi(d - \sum_{j \in S^r} s_j) = \pi d + \pi^r \sum_{j \in S^r} s_j = \pi d + \pi^r d^r$$

- Compact reformulation of SPaC, can be linearised using (8):
$$\min \left\{ \pi d + \pi^r d^r \ : \ (\pi, \pi^r) \in \operatorname{argmax} \{ (18)\text{–}(22) \} \right\}$$

- Easy to write as MPCC using (1)–(22) + their complementary slackness

Case of elastic demand

$$\min \pi \sum_{i \in B} b_i - \pi^r d^r \tag{23}$$

$$0 \leq s_j \leq sq_j \qquad j \in S \tag{24}$$

$$0 \leq b_i \leq bq_i \qquad i \in B \tag{25}$$

$$\sum_{j \in S} s_j = \sum_{i \in B} b_i \tag{26}$$

$$\sum_{j \in S^r} s_j \leq d^r \leq \sum_{i \in B} sq_i \tag{27}$$

$$\mu_i + \pi \geq bp_i \quad , \quad \mu_i \geq 0 \qquad i \in B \tag{28}$$

$$\eta_j + \pi^r - \pi \geq -sp_j \quad , \quad \eta_j \geq 0 \qquad j \in S^r \tag{29}$$

$$\eta_j - \pi \geq -sp_j \quad , \quad \eta_j \geq 0 \qquad j \in S^g \tag{30}$$

$$\sum_{i \in B}(bp_i b_i - \mu_i bq_i) \geq \sum_{j \in S}(\eta_j sq_j + sp_j s_j) \tag{31}$$

$$\pi^r(d^r - \sum_{j \in S^r} s_j) = 0 \quad , \quad pi^r \geq 0 \tag{32}$$

$$\eta_j(sq_j - s_j) = 0 \qquad j \in S \tag{33}$$

$$\mu_i(bq_i - b_i) = 0 \qquad i \in B \tag{34}$$

$$(\eta_j + \pi^r - \pi + sp_j)s_j = 0 \qquad j \in S^r \tag{35}$$

$$(\eta_j - \pi + sp_j)s_j = 0 \qquad j \in S^g \tag{36}$$

Case of elastic demand and (DC) network constraints

$$\min \ \pi \textstyle\sum_{i \in B} b_i - \pi^r d^r \tag{23}$$

$$(24), (27), (25), (26), (28), (32), (33), (34), (37) \tag{39}$$

$$m_l \leq \textstyle\sum_{k \in \mathcal{K}} S^k_l \big(\textstyle\sum_{i \in I(k)} b_i - \textstyle\sum_{j \in J(k)} s_j \big) \leq M_l \qquad l \in \mathcal{L} \tag{40}$$

$$\pi^k = \pi + \textstyle\sum_{l \in \mathcal{L}} S^k_l (\lambda^+_l - \lambda^-_l) \qquad k \in \mathcal{K} \tag{41}$$

$$\eta_j + \pi^r - \pi^{k(j)} \geq -sp_j \ , \ \eta_j \geq 0 \qquad j \in S^r \tag{42}$$

$$\eta_j - \pi^{k(j)} \geq -sp_j \ , \ \eta_j \geq 0 \qquad j \in S^g \tag{43}$$

$$\textstyle\sum_{i \in B} (bp_i b_i - \mu_i bq_i) - \textstyle\sum_{j \in S} (\eta_j sq_j + sp_j s_j) \geq \textstyle\sum_{l \in \mathcal{L}} (M_l \lambda^+_l - m_l \lambda^-_l) \tag{44}$$

$$(\eta_j + \pi^r - \pi^{k(j)} + sp_j) s_j = 0 \qquad j \in S^r \tag{45}$$

$$(\eta_j - \pi^{k(j)} + sp_j) s_j = 0 \qquad j \in S^g \tag{46}$$

$$\lambda^-_l \big(\textstyle\sum_{k \in \mathcal{K}} S^k_l \big(\textstyle\sum_{i \in I(k)} b_i - \textstyle\sum_{j \in J(k)} s_j \big) - m_l \big) = 0 \qquad l \in \mathcal{L} \tag{47}$$

$$\lambda^+_l \big(M_l - \textstyle\sum_{k \in \mathcal{K}} S^k_l \big(\textstyle\sum_{i \in I(k)} b_i - \textstyle\sum_{j \in J(k)} s_j \big) \big) = 0 \qquad l \in \mathcal{L} \tag{48}$$

$$\lambda^+_l \geq 0 \ , \ \lambda^-_l \geq 0 \qquad l \in \mathcal{L} \tag{49}$$

Outline

The algorithmic aspects

- MPCC in general $\mathcal{NP}$-hard, market clearing has to be "quick"
- Routinely done already in practice: Italian PUN, complex offers, ...
- SPaC not fundamentally more difficult than most practical EU markets, MIP-ing complementarity OK because variables nicely bounded
- Besides, when d^r is fixed it $\approx$ boils down to the original clearing problem (an LP if that was, $\approx$ whatever is currently being solved otherwise)
- Trivial approach: (cleverly) finitely sample d^r, return best solution found embarrassingly parallel (MOs can surely buy some large enough server)
- Possibly Benders' style approach (but subproblem may not be convex)
- Typical problem optimization people loves to deal with, I'd be rather optimistic we can crack it if the interest is there

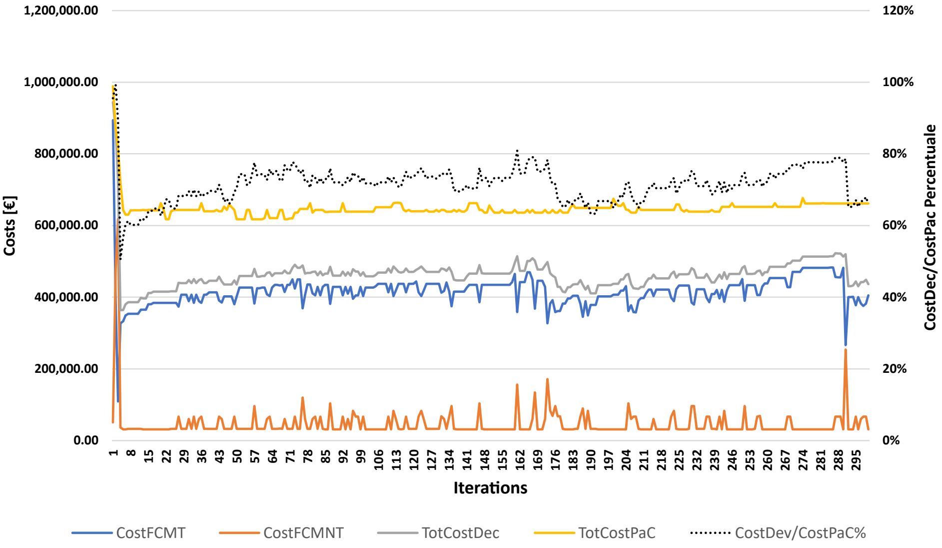
Can it be gamed?

- Of course it can: everyone offers the same (collusion)
- Bad case: all bids on g-market same as PaC, all on the r-market $\pi^\star - \varepsilon$ $\Longrightarrow$ $\pi^g = \pi^*$, $\pi^r = \pi^* - \varepsilon$ $\equiv$ negligible decrease of total system cost
- However, this reeks of collusion three miles off
- A result is proven in the paper that roughly speaking says: if enough bids in the r-market are "fair" then strategic bidders in the r-market can only achieve a fraction of π^*-PaC that decreases as $d^r \rightarrow d$ (the size of the r-market increase)
- Complicated, but: if $d^r = 0.8d$ + enough bids in S^r "low", then cost on r-market $\leq 33\%$ of π^*−PaC $\equiv$ large decrease of system cost
- Many ifs and buts, but it does seem to indicate: you need a rather serious collusion to neuter the effect

Would it work in practice?

- Hard to say, can try to get clues by Agent-Based simulations
- Simple rules to emulate behaviour of (not-too-smart) rational players:
  - if my offer was only partly accepted I very likely stay put
  - if my offer was totally accepted I may (not too likely) increase it
  - if my offer was rejected I will likely decrease it
  - if my offer is rejected for k consecutive rounds I will surely decrease it
  - anyway I will never offer below my baseline (CAPEX + OPEX) realistic cost (wind, solar, ROR hydro, hydro, coal, CGT, gas turbines, . . .)
- Tested with demand a varying fraction of $d^{\max}$ (high/low demand hours)
- Lots of parameters, set with common sense (Fabrizio knows) + minimal tuning (don't want to be cherry-picking your agents)
- Not a proof by all means, but an accepted way to get some clues

AB simulations results I



- System costs for the 30-agents test case with $d = 60\% d^{\max}$

AB simulations results II

$d\ /\ d^{max}$	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%
π^r	73.29	85.12	96.53	97.19	100.09	100.77	104.77	106.16	111.06	110.69
π^g	122.57	123.43	132.05	139.85	145	147.66	150.83	153.19	158.14	164.31
π^{PaC}	74.74	122.22	131.67	139.47	144.79	147.43	150.7	152.98	157.79	164.02
$C(S^r)/C(S^g)$	97.487	31.25	6.044	3.01	2.03	1.508	1.238	1.026	0.899	0.753
TC_SPaC/TC_PaC	98.36%	70.24%	76.19%	75.40%	76.99%	78.26%	80.48%	81.74%	83.44%	82.88%
Min	74.0%	66.4%	71.0%	70.8%	72.7%	74.9%	77.0%	78.2%	79.0%	78.2%
Max	101.5%	100.7%	99.7%	99.3%	98.7%	99.6%	100.4%	100.0%	99.1%	100.3%
Std	3.7%	3.3%	2.7%	2.7%	2.6%	2.3%	2.9%	2.9%	2.9%	3.3%

- Sample results with 100 agents (other similar except with 6, too few)
- Variable $d\ /\ d^{max}$ simulates demand fluctuation over day
- Short-term price signal still there ($\Longrightarrow$ long-term one)
- Consistent reduction in total cost save for very low demand
- Quite stable results (low std)

AB simulations results takeaways

- System does reach some sort of (realistic?) equilibrium
- Agents correctly learn how to exploit different demand scenarios
- Long- and short-term price signals on π^g conserved ($\approx$ PaC)
- S^r producers still more than decently retributed (realistic prices), just not as much as S^g producers (makes sense)
- Significant total system cost reductions (wish I could have 0.001% ...), yet not unrealistic one (historical bids gives $>$ 80%, had tell a referee)
- All in all, surprisingly (too?) reasonable results

Results with a reasonable simulation of the Italian market

- Want to see what the results would be for a realistic simulation
- Have lots of technical information available
- More sophisticated AB logic to cover different producers' behaviour
- And the results are ...

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JUST KIDDING

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JUST KIDDING

- Work just started, it'll take a while
- Doing in our spare time, not much of an help from anyone
- But we will get there, because we are curios (and we should be)

Outline

Will it ever be used in practice?

Will it ever be used in practice?

- I proposed it when the European Community asked suggestions for the highly necessary and urgent energy market reform
- Simpler approaches used in practice (the Spanish way: the State pays)
- Current "reform" based on mandatory CFD (not going to work . . .)
- Has been presented, I have written articles and been interviewed
- Just discussed in a Confindustria venue
- Would require changing the CACM, huge technical/political issues
- Whatever will happen, the story is just to start the discussion: what are the main roadblocks for optimization-based ideas to become reality?

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