



Energy Innovation Summit

Modeling Meets Optimization

The Grid Optimization Competition

Security-Constrained ACOPF
and Unit Commitment with
Gurobi

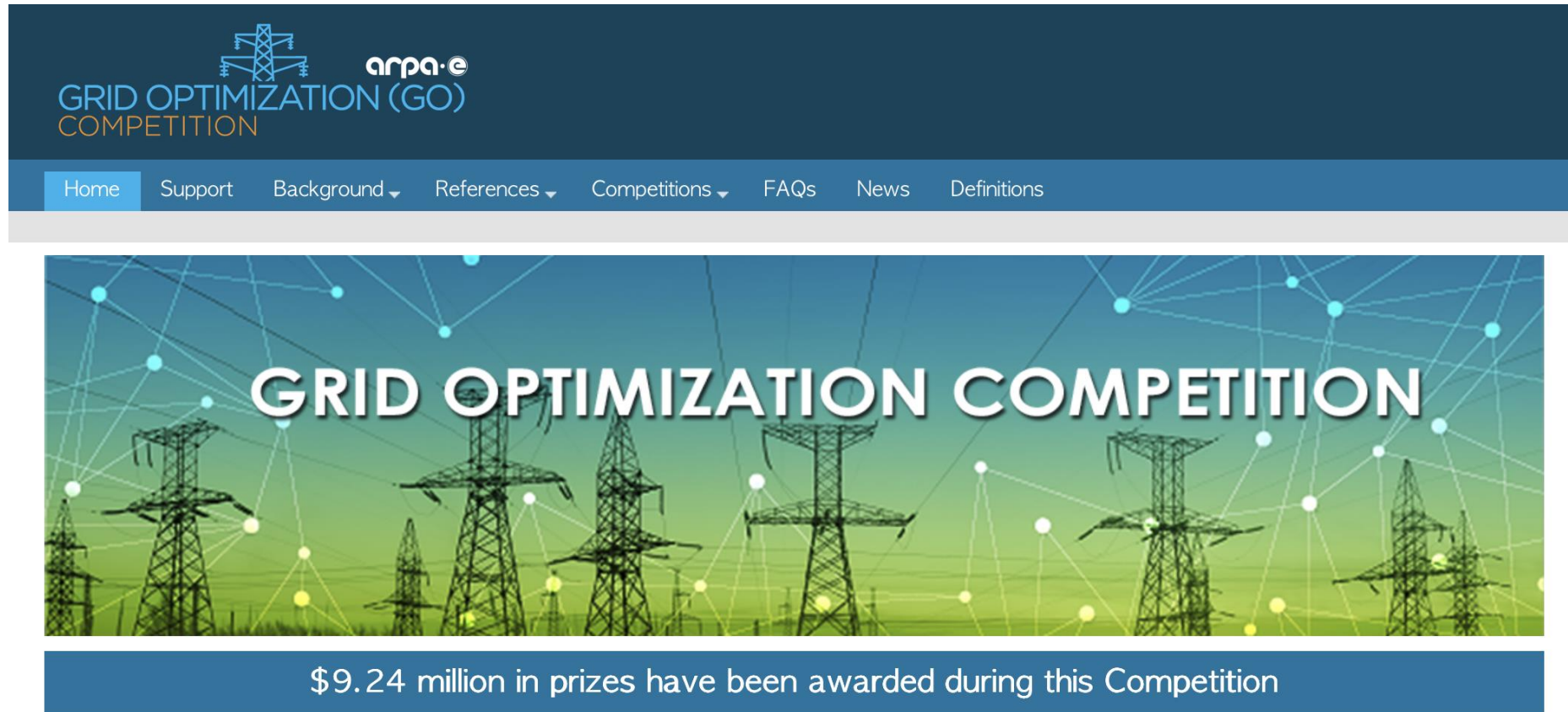
Hassan Hijazi
Senior Developer



Agenda

- Overview of the ARPA-E Grid Optimization Competition
- My personal journey in the Competition
- Recent Gurobi Improvements
- Future Gurobi Improvements
- Q&A

ARPA-E: Advanced Research Projects Agency–Energy



ARPA-E
GRID OPTIMIZATION (GO)
COMPETITION

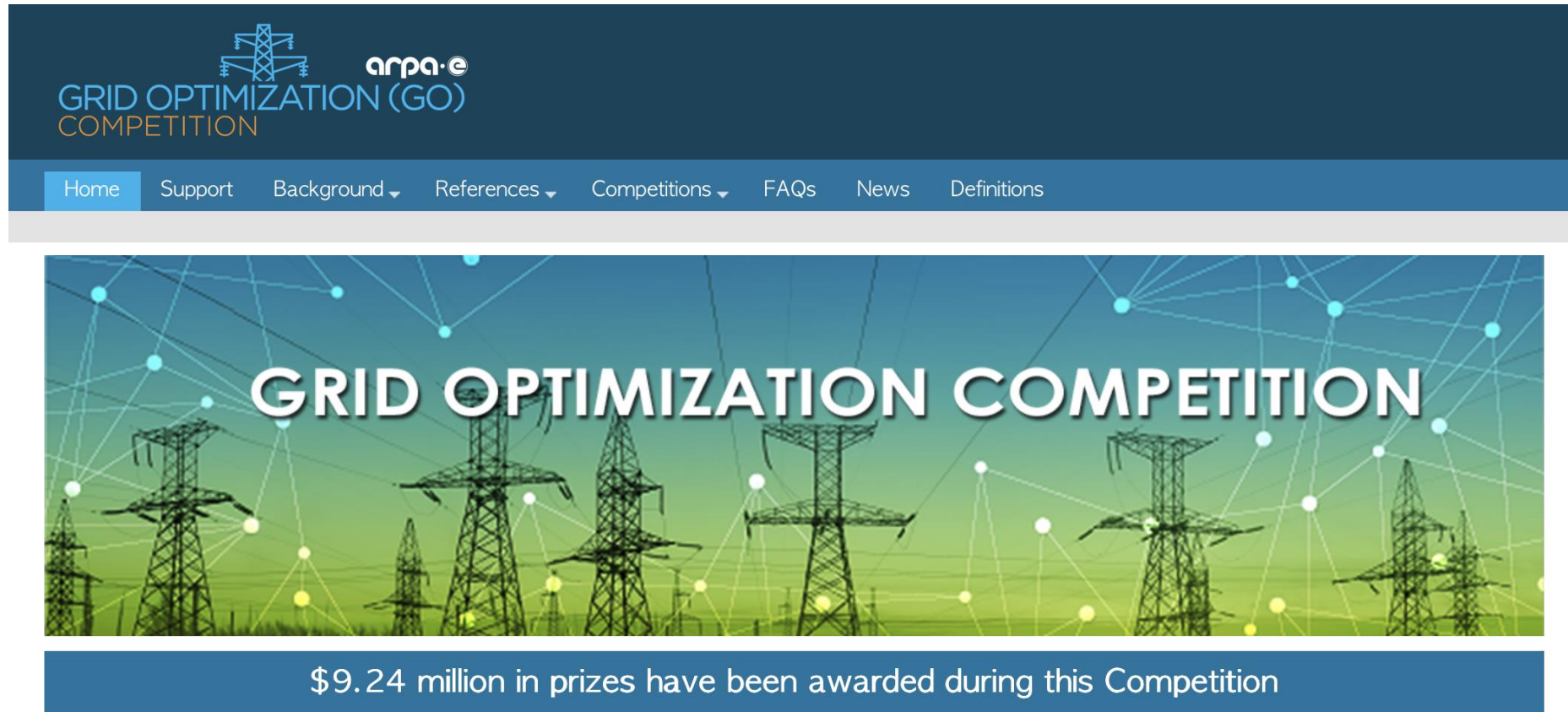
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GRID OPTIMIZATION COMPETITION

\$9.24 million in prizes have been awarded during this Competition

<https://gocompetition.energy.gov>

ARPA-E: Advanced Research Projects Agency–Energy



The image shows a screenshot of the ARPA-E Grid Optimization (GO) Competition website. At the top, there is a dark blue header with the ARPA-E logo and the text "GRID OPTIMIZATION (GO) COMPETITION". Below the header is a navigation menu with links for Home, Support, Background, References, Competitions, FAQs, News, and Definitions. The main banner features a background image of power lines and towers with a network overlay, and the text "GRID OPTIMIZATION COMPETITION" in large white letters. Below the banner, a dark blue bar contains the text "\$9.24 million in prizes have been awarded during this Competition".

GRID OPTIMIZATION (GO)
COMPETITION

Home Support Background References Competitions FAQs News Definitions

GRID OPTIMIZATION COMPETITION

\$9.24 million in prizes have been awarded during this Competition

<https://gocompetition.energy.gov>

Ran from 2015 to 2024
Longest ARPA-E project in history

My Personal Journey in the Competition

The Competitors



Challenge 1 (2019-2020)

26 teams

Security-Constrained ACOPF

Continuous model, solution still feasible in the event of the loss of any single power line or generator in the system



(6th, 6th, 5th, 4th)

Participants upload binaries/source code on 

Code evaluated on hidden datasets

Datasets published after each Trial

| | Lowest Cost | Performance Profiles |
|--------------------|-------------------------------------|-------------------------------------|
| Real-Time (10 Min) | Division 1 Top 10: \$100k | Division 3 Top 10: \$100k |
| Offline (45 Min) | Division 2 Top 10: \$100k | Division 4 Top 10: \$100k |

Figure 2. Division Breakdown and Scoring Approach for Challenge 1 Awards.

Challenge 2 (2020-2021)

16 teams

Security-Constrained Unit Commitment
ACOPF (single time step)

New challenges:

- Adjustable transformer tap ratios (discrete)
- Phase shifting transformers (non-convex)
- Switchable lines and shunts (discrete)
- Price-responsive demand (more variables)
- Ramping generators and loads (more constraints)
- Fast-start unit commitment (discrete)



(1st, 1st, 1st, 1st)

| Transmission Line and Transformer Switching NOT Allowed | | Transmission Line and Transformer Switching Allowed | |
|---|---|---|---|
| Real-Time (5 Min) | Offline (60 Min) | Real-Time (5 Min) | Offline (60 Min) |
| Division 1 1 st place: \$150k 2 nd place: \$120k 3 rd place: \$90k 4 th place: \$60k 5 th place: \$30k | Division 2 1 st place: \$150k 2 nd place: \$120k 3 rd place: \$90k 4 th place: \$60k 5 th place: \$30k | Division 3 1 st place: \$150k 2 nd place: \$120k 3 rd place: \$90k 4 th place: \$60k 5 th place: \$30k | Division 4 1 st place: \$150k 2 nd place: \$120k 3 rd place: \$90k 4 th place: \$60k 5 th place: \$30k |
| Greatest Market Surplus | | | |

Monarch of the Mountain (2021-2022)

6 teams

Offline (no time limits)

Public Instances (no secret datasets)

- \$5,000 award per scenario if objective is improvement is $> 1\%$ compared to Challenge 2
- \$5,000 per scenario if top ranked for the greatest number of competition weeks



1st

| Submitting Team | Total Submissions | Best Result | Longest | $> 1\%$ | Prizes |
|------------------|-------------------|-------------|---------|---------|-----------|
| GravityX | 148 | 74 | 82 | 4 | \$430,000 |
| Gordian Knot | 18 | 2 | 2 | | \$10,000 |
| GOT-BSI-OPF | 22 | 8 | | | |
| Monday Mornings | 36 | | | | |
| CasePower | 12 | | | | |
| Artelys_Columbia | 1 | | | | |
| Total | 201 | 84 | 84 | 4 | \$440,000 |

Challenge 3 (2022-2023)

18 teams

Security-Constrained Unit Commitment ACOPF with
Transmission Switching
(multiperiod)



New challenges:

- Temporal Constraints (18 to 48 time-steps)
- $1e-8$ Constraint Satisfaction (4 orders of magnitude drop!)
- Dense Reserve Constraints (thousands of nnz in one constraint)

Challenge 3 (2022-2023)

18 teams

Security-Constrained Unit Commitment ACOPF with
Transmission Switching
(multiperiod)



Real-Time Market (10 Min) **Day-Ahead Market (120 Min)** **Week Ahead Advisory (240 Min)**

Division 1
1st place: \$100k
2nd place: \$90k
3rd place: \$80k
4th place: \$70k
5th place: \$60k

Division 2
1st place: \$100k
2nd place: \$90k
3rd place: \$80k
4th place: \$70k
5th place: \$60k

Division 3
1st place: \$100k
2nd place: \$90k
3rd place: \$80k
4th place: \$70k
5th place: \$60k



Cumulative Objective Function

Real-Time Market (10 Min) **Day-Ahead Market (120 Min)** **Week Ahead Advisory (240 Min)**

Division 4
1st place: \$100k
2nd place: \$90k
3rd place: \$80k
4th place: \$70k
5th place: \$60k

Division 5
1st place: \$100k
2nd place: \$90k
3rd place: \$80k
4th place: \$70k
5th place: \$60k

Division 6
1st place: \$100k
2nd place: \$90k
3rd place: \$80k
4th place: \$70k
5th place: \$60k



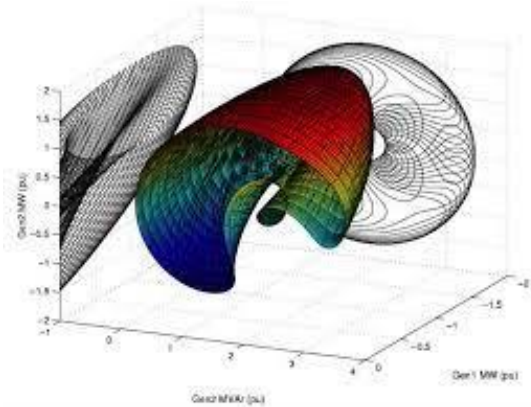
Number of best scores

(6th, 5th, 9th)
(3rd, 2nd, 2nd)

The ARPA-E Grid Optimization Competition

At an abstract level: a large-scale MINLP

Many many non-convex constraints!



$$\min f(x, y)$$

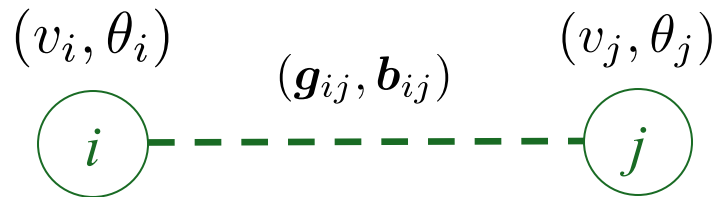
$$s.t. g_1(x, y) \leq 0$$

...

$$g_m(x, y) \leq 0$$

$$x \in \mathbb{R}^n, y \in \mathbb{Z}^q$$

Many many discrete and continuous variables!



$$p_{ij} = g_{ij} v_i^2 - v_i v_j (g_{ij} \cos(\theta_i - \theta_j) + b_{ij} \sin(\theta_i - \theta_j))$$

AC Optimal Power Flow

A complete mathematical model is as follows,

variables:

$$S_k^g \quad \forall k \in G - \text{generator complex power dispatch} \quad (1)$$

$$V_i \quad \forall i \in N - \text{bus complex voltage} \quad (2)$$

$$S_{ij} \quad \forall (i, j) \in E \cup E^R - \text{branch complex power flow} \quad (3)$$

$$\text{minimize: } \sum_{k \in G} c_{2k} (\Re(S_k^g))^2 + c_{1k} \Re(S_k^g) + c_{0k} \quad (4)$$

subject to:

$$\angle V_r = 0 \quad \forall r \in R \quad (5)$$

$$S_k^{gl} \leq S_k^g \leq S_k^{gu} \quad \forall k \in G \quad (6)$$

$$v_i^l \leq |V_i| \leq v_i^u \quad \forall i \in N \quad (7)$$

$$\sum_{k \in G_i} S_k^g - \sum_{k \in L_i} S_k^d - \sum_{k \in S_i} (Y_k^s)^* |V_i|^2 = \sum_{(i,j) \in E_i \cup E_i^R} S_{ij} \quad \forall i \in N \quad (8)$$

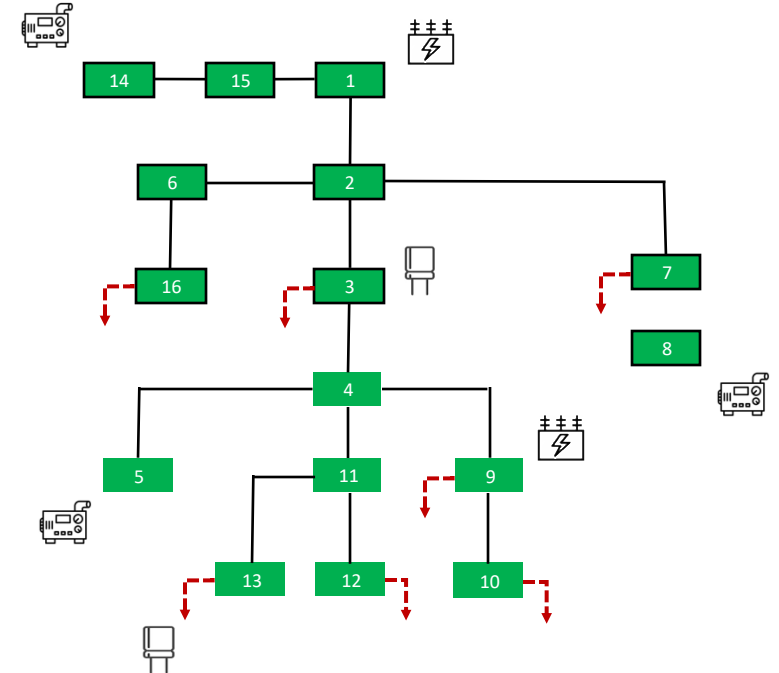
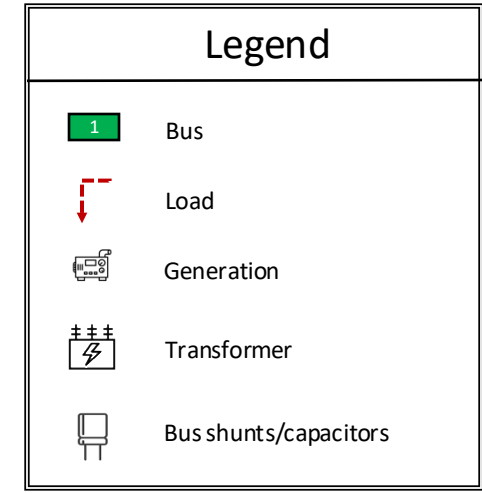
$$S_{ij} = (Y_{ij} + Y_{ij}^c)^* \frac{|V_i|^2}{|T_{ij}|^2} - Y_{ij}^* \frac{V_i V_j^*}{T_{ij}} \quad \forall (i, j) \in E \quad (9)$$

$$S_{ji} = (Y_{ij} + Y_{ji}^c)^* |V_j|^2 - Y_{ij}^* \frac{V_i^* V_j}{T_{ij}^*} \quad \forall (i, j) \in E \quad (10)$$

$$|S_{ij}| \leq s_{ij}^u \quad \forall (i, j) \in E \cup E^R \quad (11)$$

$$|S_{ij}| \leq |V_i| i_{ij}^u \quad \forall (i, j) \in E \cup E^R \quad (12)$$

$$\theta_{ij}^{\Delta l} \leq \angle(V_i V_j^*) \leq \theta_{ij}^{\Delta u} \quad \forall (i, j) \in E \quad (13)$$

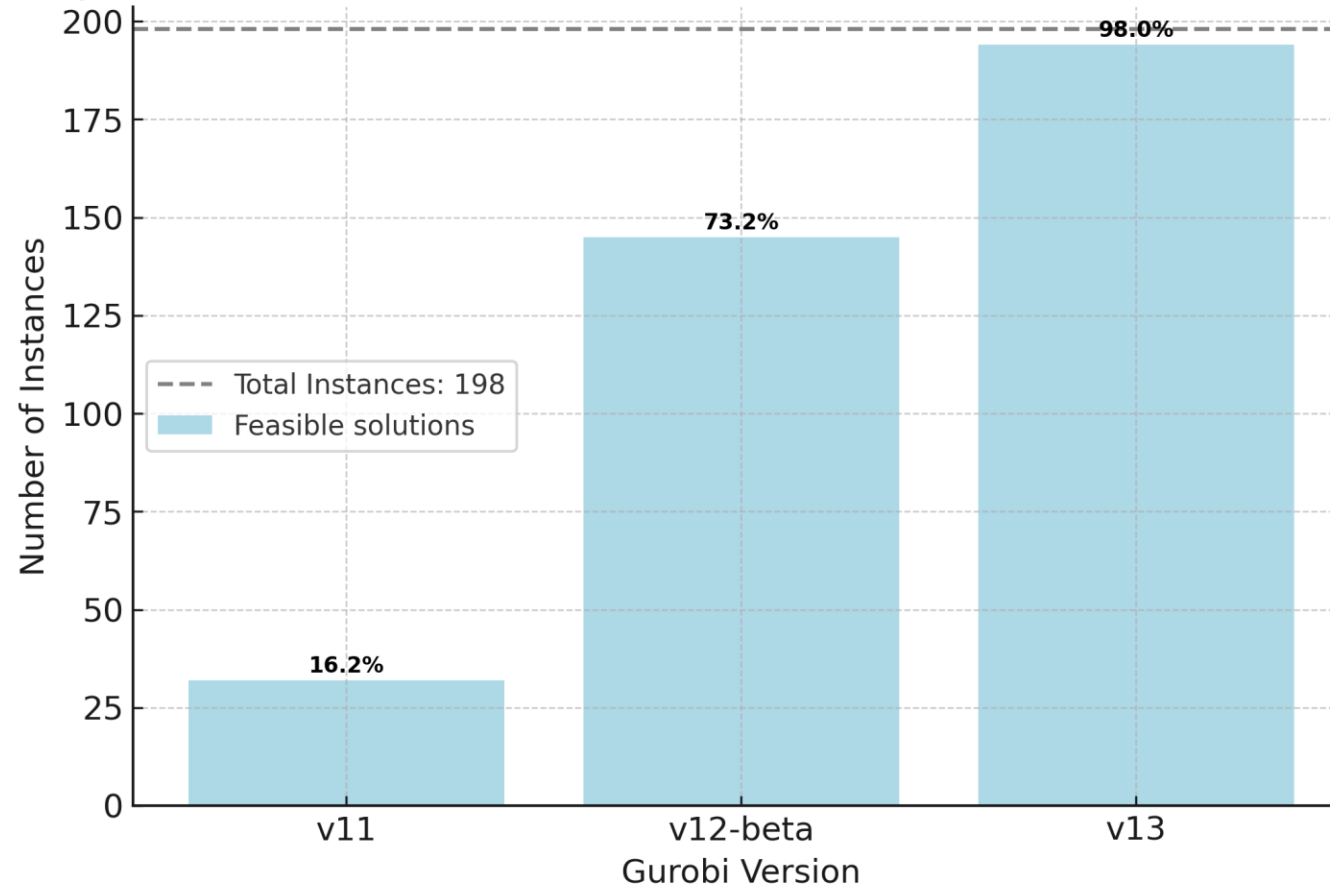


Recent Gurobi Improvements

- ACOPF Gurobi Optimods
- DCOPF Topology Optimization

ACOPF (QCQP formulation)

ACOPF OptiMods on PGLIB Benchmark (total of 198 instances, TimeLimit=600)



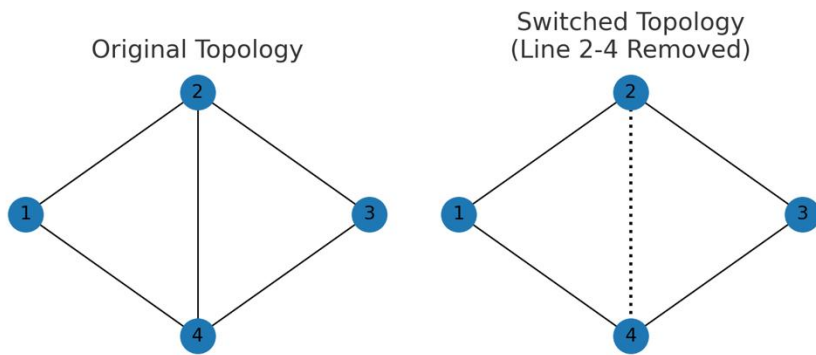
ACR: rectangular formulation (QCQP)

PGLIB Benchmarks: <https://github.com/power-grid-lib/pglib-opf>

- Fixed the QCQP formulation
- Better handling of starting points
- Restoration Feasibility

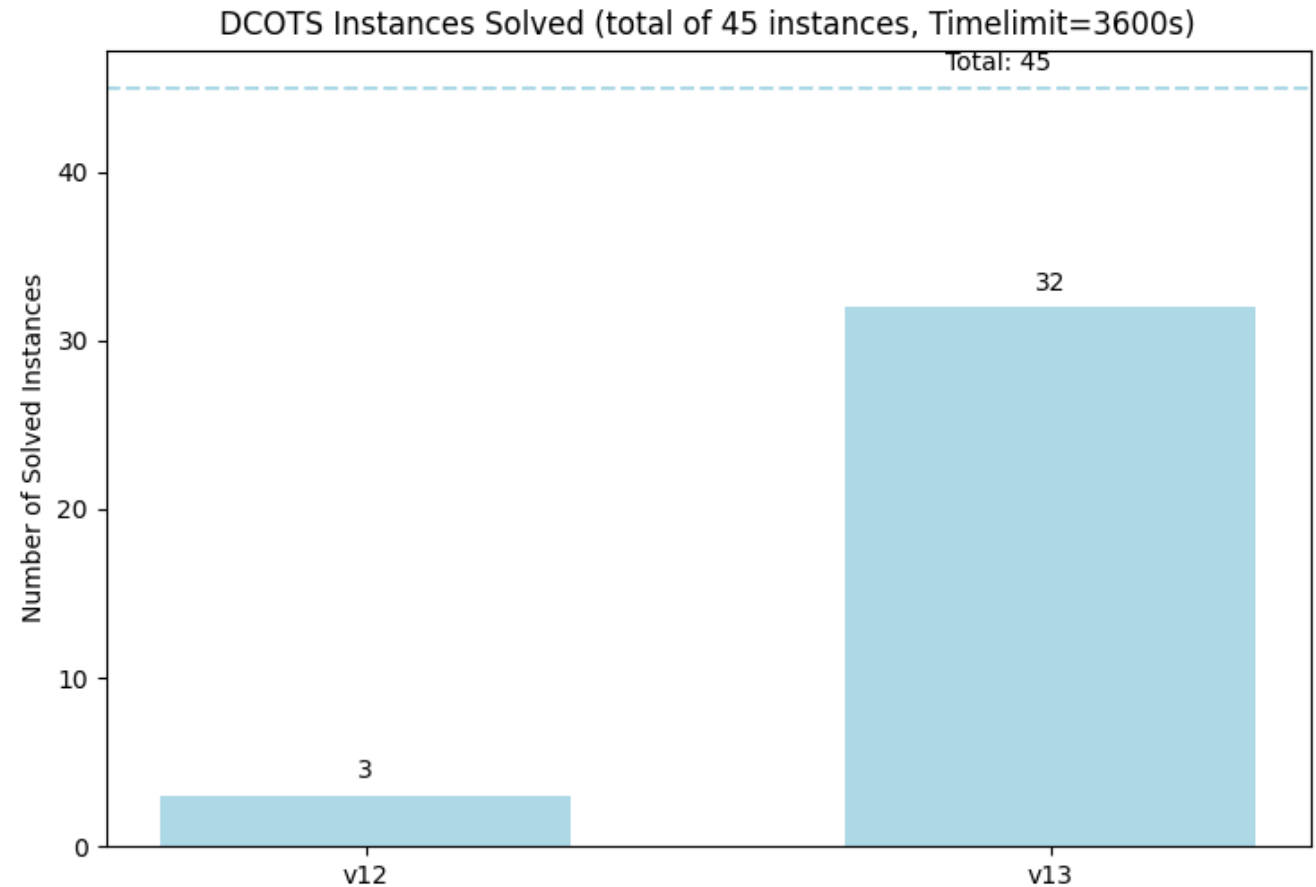
DCOPF Topology Optimization

$$0 \leq p_{ij} \leq Mz_{ij}$$



Why Topology Optimization Matters:

- Failure Recovery
- Reduce stress on congested lines (Germany spent EUR 4 billion on Congestion Management in 2022*)



- On/Off detection and heuristic

* <https://www.iea.org/commentaries/grid-congestion-is-posing-challenges-for-energy-security-and-transitions>

Future Gurobi Improvements

- Focus on Important Variables
- Detecting Structure and Batching Variables
- Rolling Horizon Decomposition
- Iterative NLP Refinement

Ingredient 1: Focus on What Matters

Largest instance has 1,224,080,000 variables and 837,488,000 constraints.



Many variables are weakly connected in the interaction graph or have low impact on the objective function

Due to the flexibility of contingency constraints and the large cost of base-case objective

Largest instance has 612,040 **important** variables and 418,744 **important** constraints.

Ingredient 1: Focus on What Matters

Largest instance has 1,224,080,000 variables and 837,488,000 constraints.



Many variables are weakly connected in the interaction graph or have low impact on the objective function

Generalization Idea

- Compute an indirect “Objective weight” for each variable using paths in the interaction graph leading to terms appearing in the objective
- Discard variables (and constraints they appear in) if weight is below a given threshold

Ingredient 2: Grouping Discrete Variables

Algorithm 1 Iterative Batch Rounding (IBR)

- 1: Group discrete variables into predefined batches \mathcal{B}_1 to \mathcal{B}_n .
 - 2: Solve continuous relaxation of MINLP (1).
 - 3: **for** $i \in \{1, \dots, n\}$ **do**
 - 4: Call the custom ROUND function on batch \mathcal{B}_i
 - 5: Fix all rounded variables in batch \mathcal{B}_i
 - 6: Solve the continuous relaxation of reduced MINLP (1).
 - 7: **end for**
-

Generalization Idea

- Batching strategy
- Custom rounding for each batch
- Batch ordering

Close to Fix-and-Relax [2]

[2] G. Belvaux and L. A. Wolsey, "bc—prod: A specialized branch-and-cut system for lot-sizing problems," *Management Science*, vol. 46, no. 5, pp. 724–738, 2000

Without IBR

Barrier solved model in 49 iterations and 6.52 seconds (6.52 work units)
Optimal objective 4.14182236e+06

Root relaxation: objective 4.141822e+06, 0 iterations, 5.80 seconds (6.03 work units)
Another try with MIP start

| Nodes | | Current Node | | | Objective Bounds | | | Work | |
|-------|--------|--------------|-------|--------|------------------|------------|-----|---------|------|
| Expl | Unexpl | Obj | Depth | IntInf | Incumbent | BestBd | Gap | It/Node | Time |
| 0 | 0 | 4141822.36 | 0 | 111 | - | 4141822.36 | - | - | 6s |
| 0 | 0 | 4141456.14 | 0 | 26 | - | 4141456.14 | - | - | 16s |
| 0 | 0 | 4141455.80 | 0 | 20 | - | 4141455.80 | - | - | 32s |
| 0 | 0 | 4141446.14 | 0 | 12 | - | 4141446.14 | - | - | 40s |
| 0 | 0 | 4141445.04 | 0 | 18 | - | 4141445.04 | - | - | 49s |
| 0 | 0 | 4141442.17 | 0 | 78 | - | 4141442.17 | - | - | 57s |
| 0 | 0 | 4141442.17 | 0 | 10 | - | 4141442.17 | - | - | 65s |
| 0 | 0 | 4141442.17 | 0 | 18 | - | 4141442.17 | - | - | 72s |
| 0 | 2 | 4141442.17 | 0 | 16 | - | 4141442.17 | - | - | 114s |
| 1 | 4 | 4141442.17 | 1 | 18 | - | 4141442.17 | - | 0.0 | 123s |
| 3 | 8 | 4141442.17 | 2 | 16 | - | 4141442.17 | - | 0.0 | 137s |
| 7 | 16 | 4141442.17 | 3 | 8 | - | 4141442.17 | - | 0.0 | 163s |

With IBR

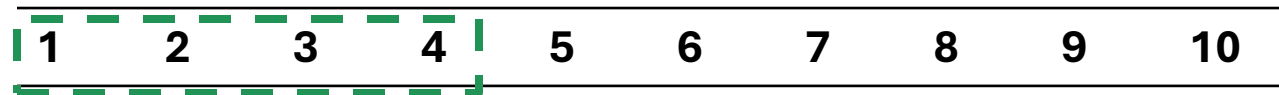
Barrier solved model in 49 iterations and 6.46 seconds (6.52 work units)

Optimal objective 4.14182236e+06

Root relaxation: objective 4.141822e+06, 0 iterations, 5.82 seconds (6.03 work units)

| Nodes | | Current Node | | | Objective Bounds | | | Work | |
|-------|--------|--------------|-------|--------|------------------|------------|-------|---------|------|
| Expl | Unexpl | Obj | Depth | IntInf | Incumbent | BestBd | Gap | It/Node | Time |
| 0 | 0 | 4141822.36 | 0 | 111 | 4141167.81 | 4141822.36 | 0.02% | - | 6s |

Ingredient 3: Rolling Horizon Decomposition



$$\left. \begin{array}{l} \min \mathbf{c}_1^t x + \mathbf{c}_2^t y \\ s.t. \mathbf{A}_1 x + \mathbf{A}_2 y \leq \mathbf{b} \\ x \in \mathbb{R}^n, y \in \mathbb{Z}^n \end{array} \right\}$$



Ingredient 3: Rolling Horizon Decomposition



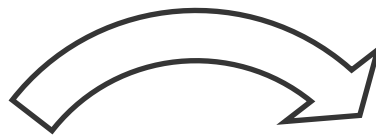
Generalization Idea

- Automatically identify multiperiod constraints/vars
- Implement a Rolling Horizon Decomposition

Ingredient 4: Iterative NLP Refinement

$$p_e^{fr} = \left(\mathbf{g}_e + \mathbf{g}_e^{fr} \right) \frac{v_i^2}{\tau_e^2} - \frac{v_i v_j}{\tau_e} \left(\mathbf{g}_e \cos(\theta_i - \theta_j - \theta_e) + \mathbf{b}_e \sin(\theta_i - \theta_j - \theta_e) \right)$$

$$p_e^{to} = \left(\mathbf{g}_e + \mathbf{g}_e^{to} \right) v_j^2 - \frac{v_i v_j}{\tau_e} \left(\mathbf{g}_e \cos(\theta_j - \theta_i + \theta_e) + \mathbf{b}_e \sin(\theta_j - \theta_i + \theta_e) \right)$$



MIP

NLP



Ingredient 4: Iterative NLP Refinement

$$p_e^{fr} = \left(\mathbf{g}_e + \mathbf{g}_e^{fr} \right) \frac{v_i^2}{\tau_e^2} - \frac{v_i v_j}{\tau_e} \left(\mathbf{g}_e \cos(\theta_i - \theta_j - \theta_e) + \mathbf{b}_e \sin(\theta_i - \theta_j - \theta_e) \right)$$

$$p_e^{to} = \left(\mathbf{g}_e + \mathbf{g}_e^{to} \right) v_j^2 - \frac{v_i v_j}{\tau_e} \left(\mathbf{g}_e \cos(\theta_j - \theta_i + \theta_e) + \mathbf{b}_e \sin(\theta_j - \theta_i + \theta_e) \right)$$

$$p_e^{fr} + p_e^{to} = \mathbf{x}_{nlp}^*$$

Generalization Idea

- Using the symbolic interaction graph, generate linear constraints with constant right-hand-side replacing complex NL functions
- Iteratively refine these right-hand-sides until convergence of the MIP-NLP decomposition

Ingredient 4: Iterative NLP Refinement

No Refinement

One-Iteration
Refinement

| Instance | Scenario | | Lossless-DC | Lossy-DC | |
|--------------|--------------|---------------|--------------|-------------|---------------|
| C3E3N01576D1 | scenario_027 | 96,227,154 | 98,057,070 | 97,780,021 | 98,298,517 |
| C3E3N04224D1 | scenario_131 | 91,168,493 | 91,141,339 | 91,142,009 | 91,169,130 |
| C3E3N06049D1 | scenario_031 | 104,152,318 | 104,095,683 | 104,093,510 | 104,135,871 |
| C3E3N06717D1 | scenario_031 | Fail | -889,688,701 | 136,130,464 | 136,589,493 |
| C3E3N08316D1 | scenario_001 | 1,158,730,927 | Fail | Fail | 1,180,957,975 |

We're maximizing market surplus \$ (higher is better)

UC-ACOPF (Challenge 2 formulation)

Public Instances:

<https://gocompetition.energy.gov/challenge-2-final-event-synthetic-data>

Current Gurobi performance on smallest instance (600 nodes, largest has 30k) with no security constraints (MIQCQP):

Presolved: **32413** rows, **17747** columns, 105702 nonzeros

Presolved model has 2928 quadratic constraint(s)

Presolved model has 7318 bilinear constraint(s)

Root relaxation: objective 7.166463e+05, 19096 iterations, 2.72 seconds (2.28 work units)

Starting root helper type 2, index 0

Starting root helper type 5, index 1

| Nodes | | Current Node | Objective Bounds | | | Work | | | Time |
|-------|--------|--------------|------------------|--------|--------------------|------------|-------|---------|---------------|
| Expl | Unexpl | Obj | Depth | IntInf | Incumbent | BestBd | Gap | It/Node | |
| 0 | 0 | 716646.261 | 0 | 5024 | -764,690.81 | 716646.261 | 194% | - | 6s |
| ... | | | | | | | | | |
| 1 | 4 | 716646.261 | 1 | 4934 | -759,239.54 | 716646.261 | 194% | 6877 | 1415s |
| H2 | 4 | | | | 357,807.45 | 716646.261 | 100% | 3438 | 1415s |
| ... | | | | | | | | | |
| 24383 | 11143 | 716646.261 | 79 | 4933 | 453,718.14 | 716646.261 | 57.9% | 3490 | 60570s |

Best competition solution has an objective value of **625,561** (computed in less than 600 seconds)



Energy Innovation Summit

Modeling Meets Optimization

Questions

<http://gocompetition.energy.gov/>

<https://github.com/lanl-ansi/PowerModelsSecurityConstrained.jl>

<https://github.com/power-grid-lib/pglib-opf/>

