The Burrito Optimization Game

Game and slides by Gurobi Optimization, in collaboration with Prof. Larry Snyder, Lehigh University

GUROBI OPTIMIZATION

Outline

1. Introduction
2. Coping with Uncertainty
3. Optimization 101

GUROBI OPTIMIZATION

Introduction

GUROBI
OPTIMIZATION

Guroble needs your help!

- Guroble has just launched its business in Burritoville
- Where to place burrito trucks to serve customers and maximize profit?
- Plan carefully:
  - Every truck has a cost
  - Revenue depends on how close to customers
- Each day brings new challenges

BURRITO©
OPTIMIZATION GAME

Click here!

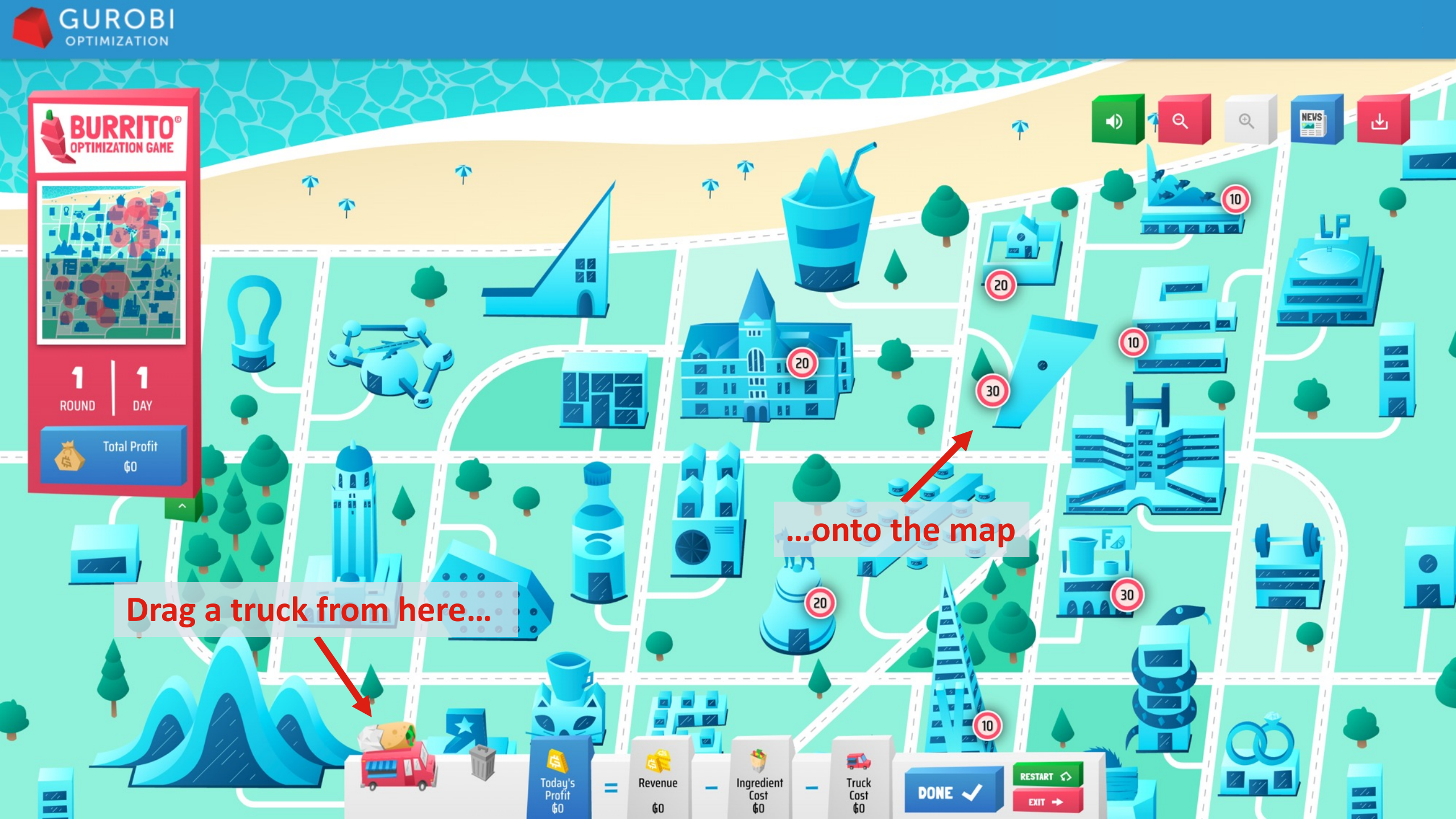
Can you find the best spots to place burrito trucks and maximize your profit? Get as close as possible to optimal placement to win.

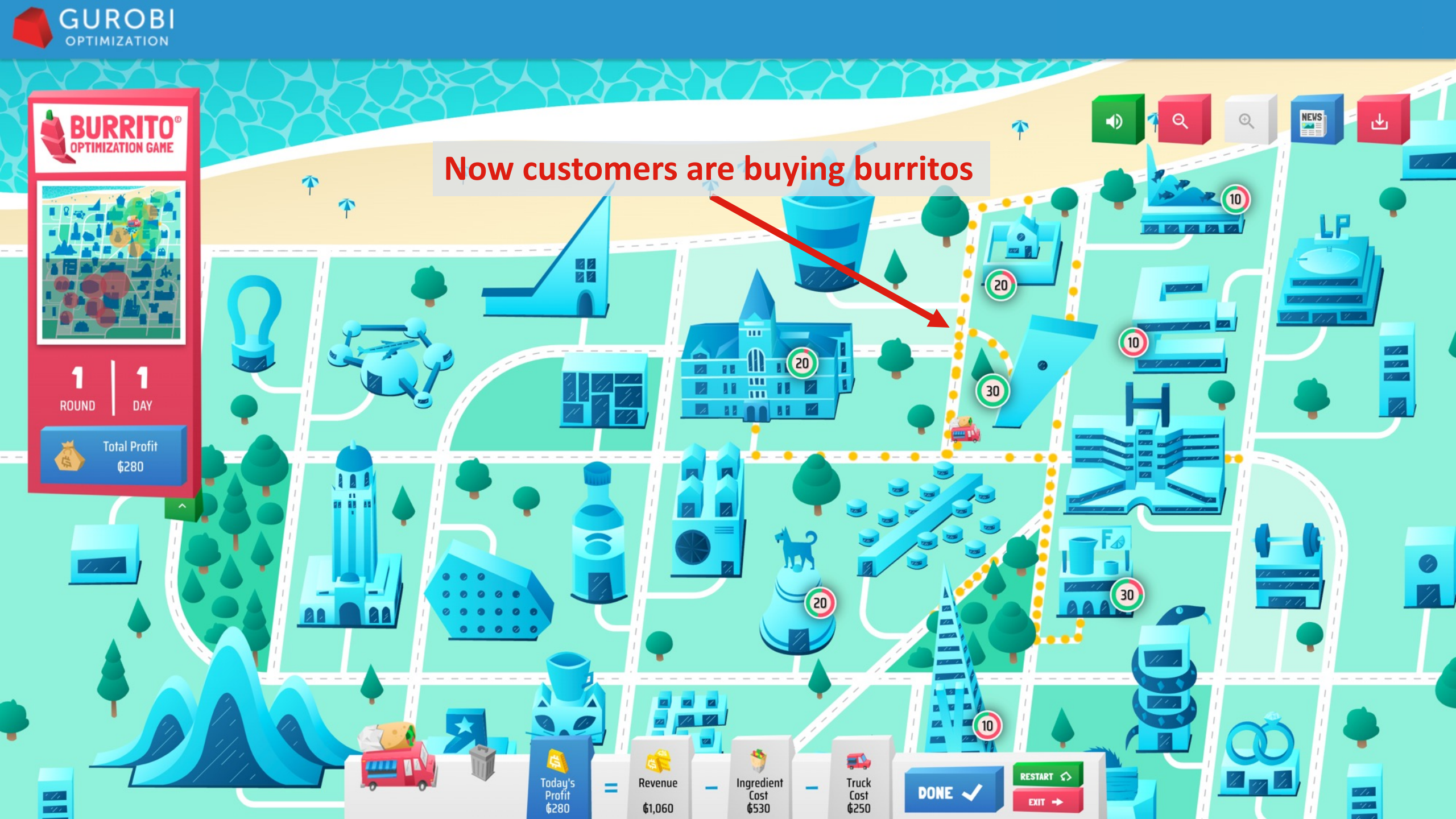
PLAY THE GAME
Play the game with increasing difficulty.

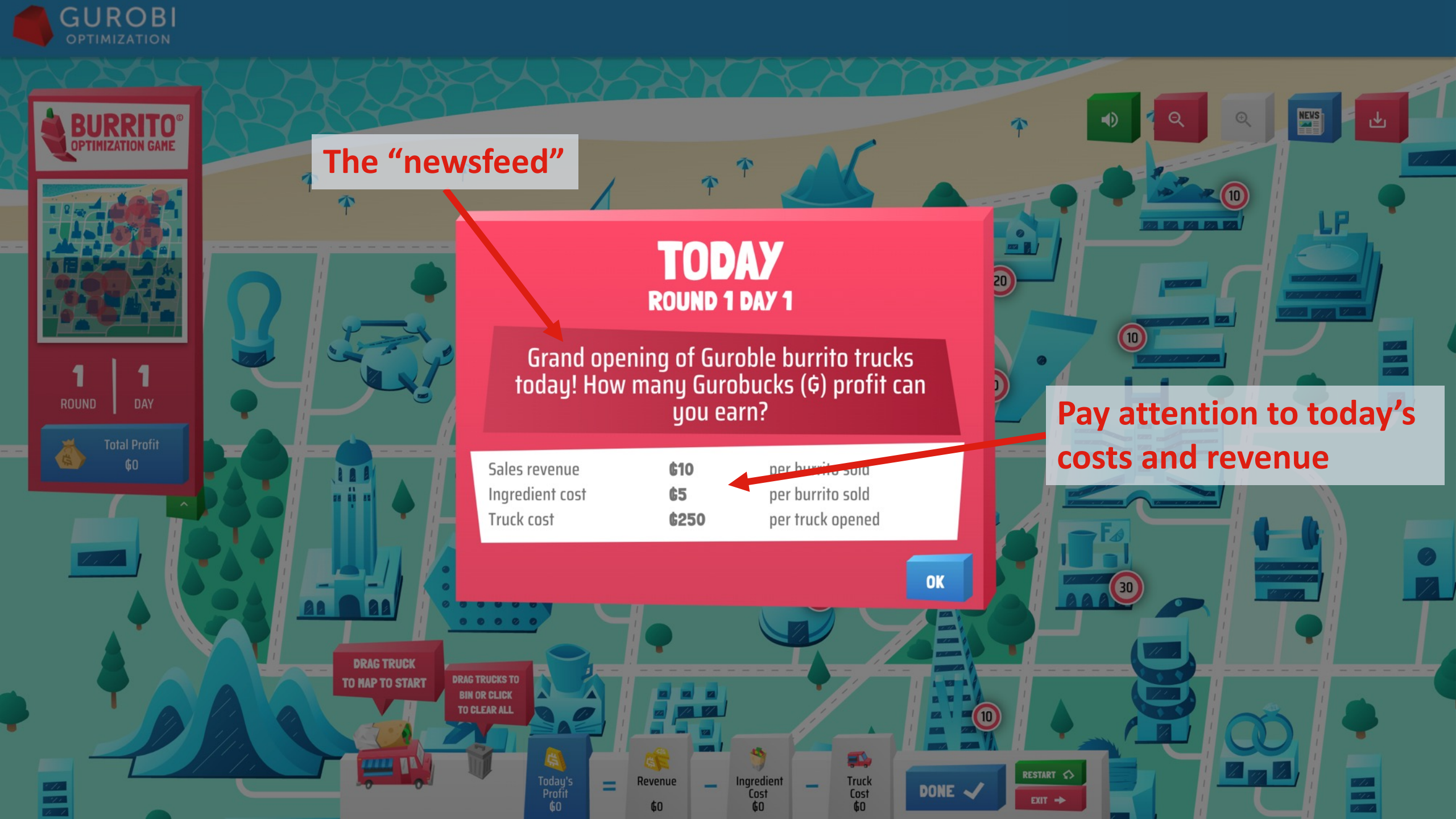
CHAMPIONSHIP MODE
Compete against participants in an event.

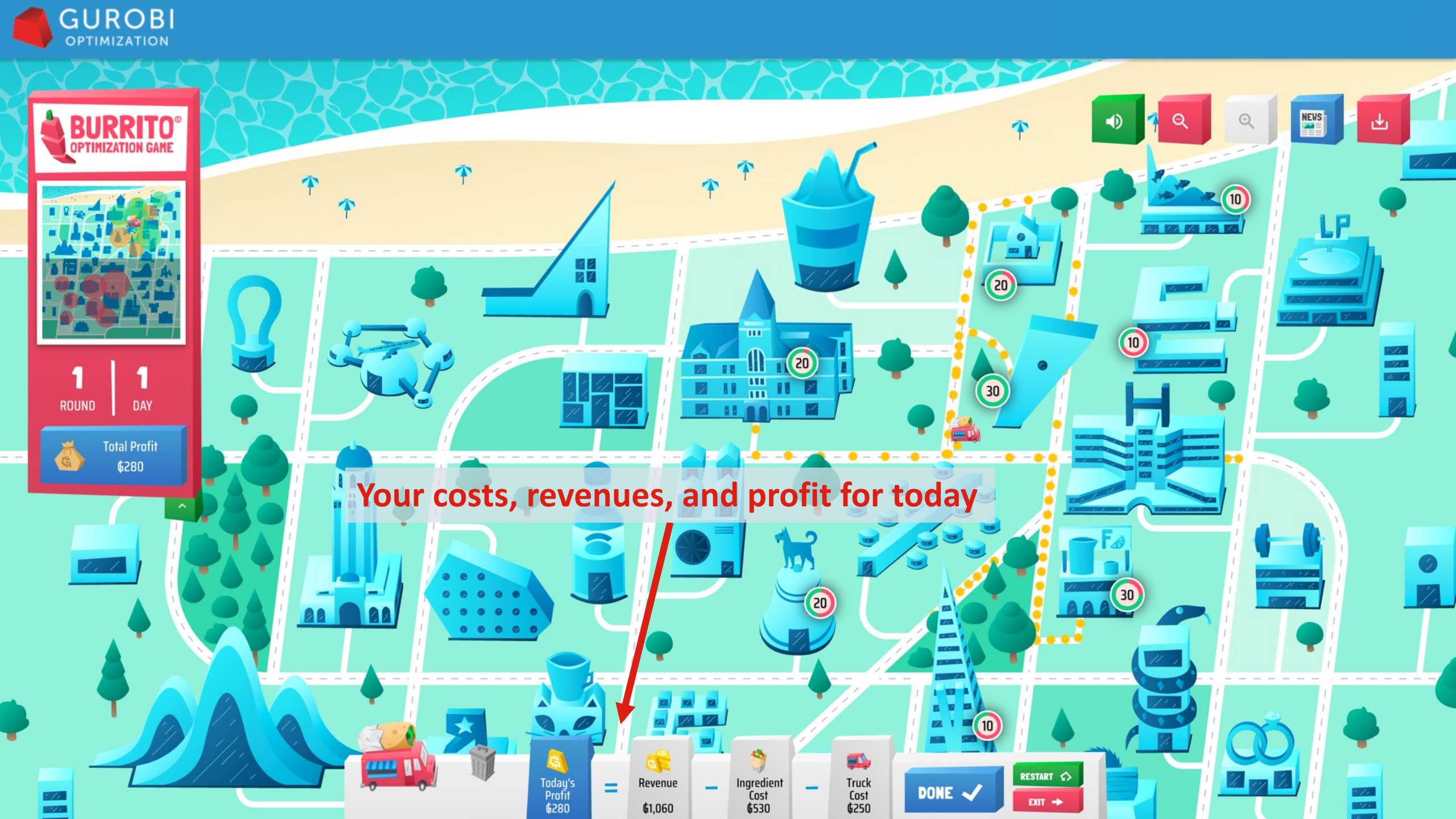
GAME GUIDE

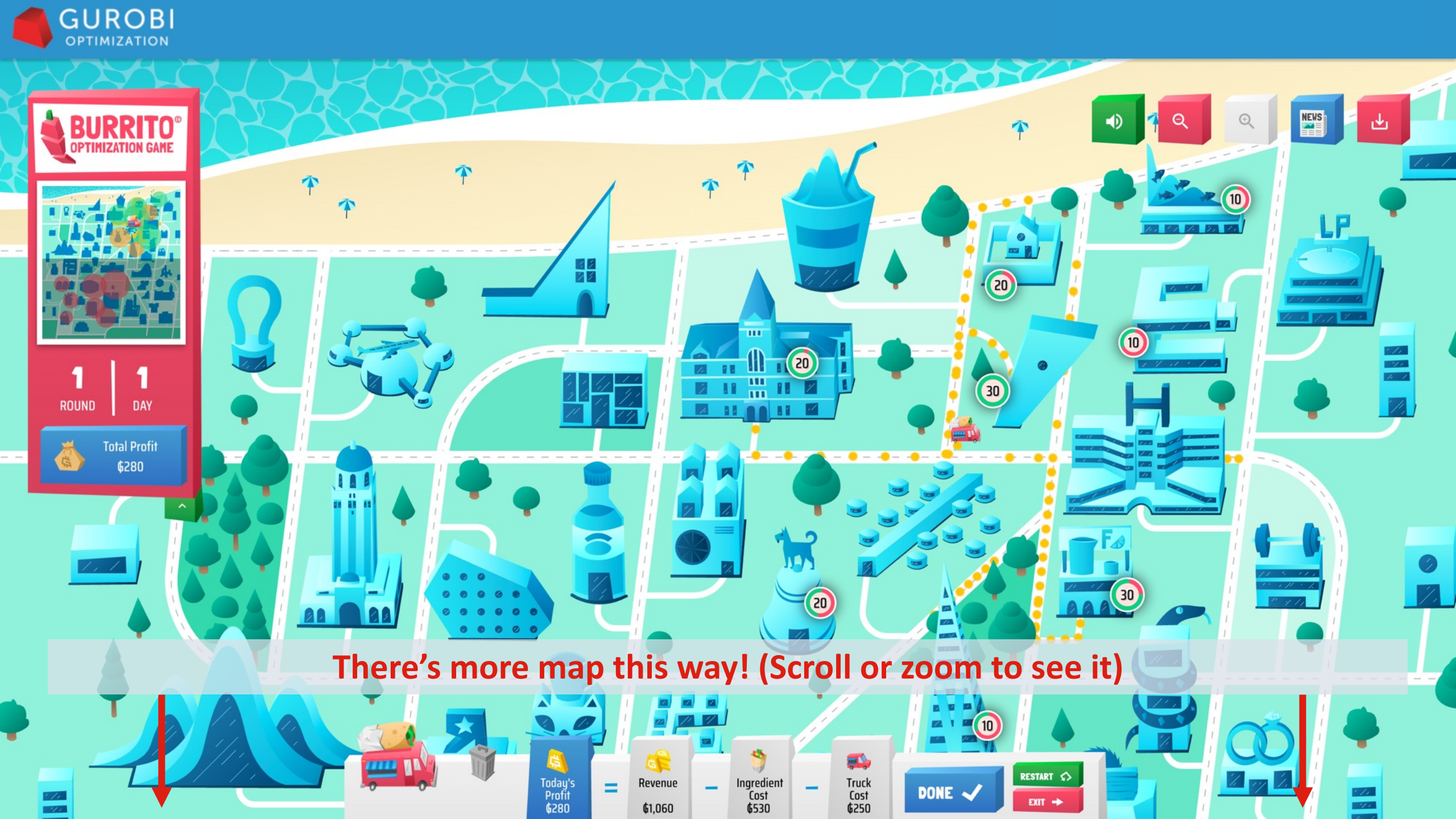
CREDITS

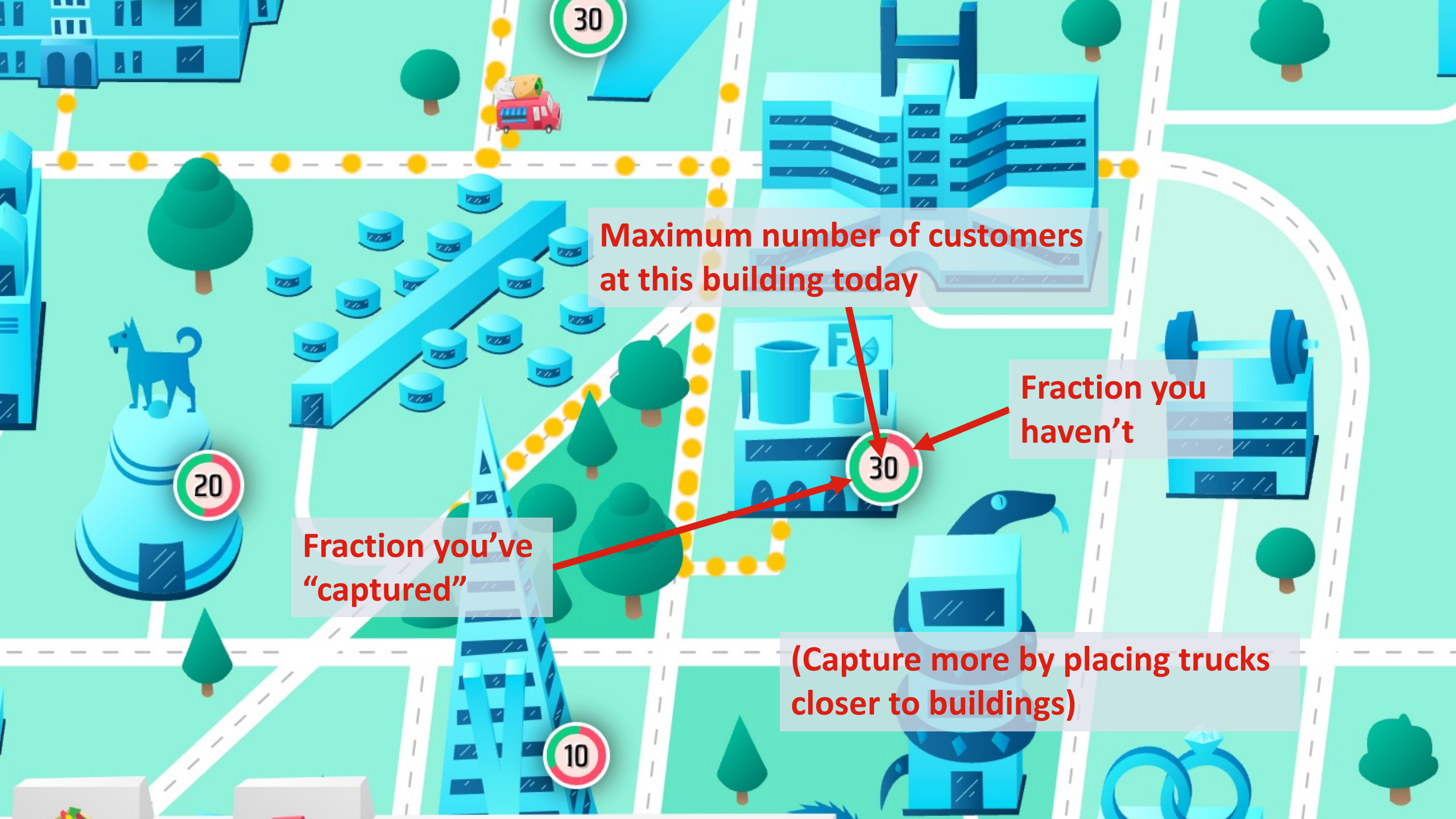

GUROBI
OPTIMIZATION
BURRITO®
OPTIMIZATION GAME
1
ROUND
1
DAY
Total Profit
¢0
NEWS
...onto the map
Drag a truck from here...
Today's Profit ¢0
=
Revenue ¢0
−
Ingredient Cost ¢0
−
Truck Cost ¢0
DONE
RESTART
EXIT

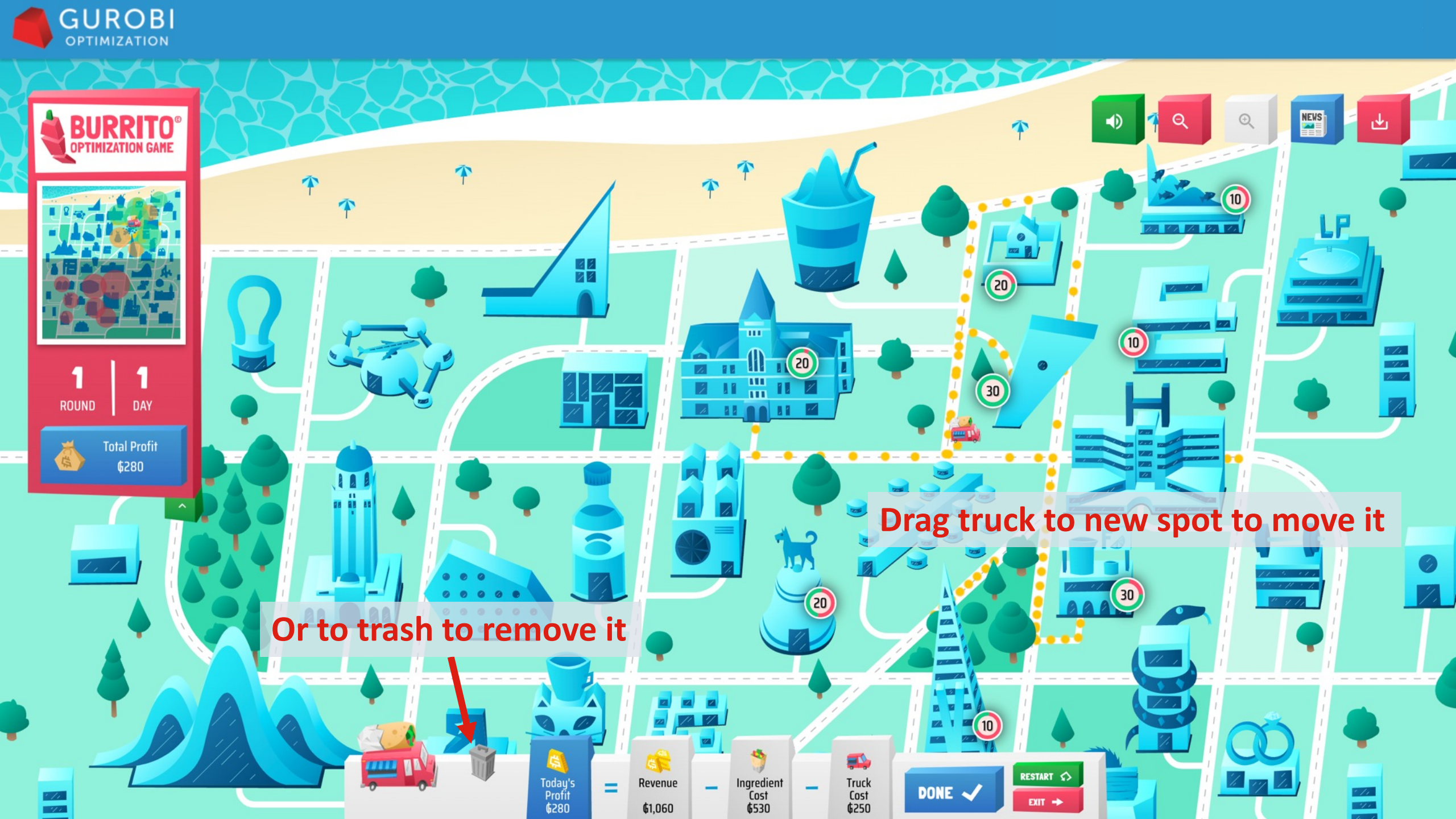

GUROBI
OPTIMIZATION
BURRITO®
OPTIMIZATION GAME
1
ROUND
1
DAY
Total Profit
₲280
Now customers are buying burritos
NEWS
Today's Profit ₲280
=
Revenue ₲1,060
−
Ingredient Cost ₲530
−
Truck Cost ₲250
DONE
RESTART
EXIT

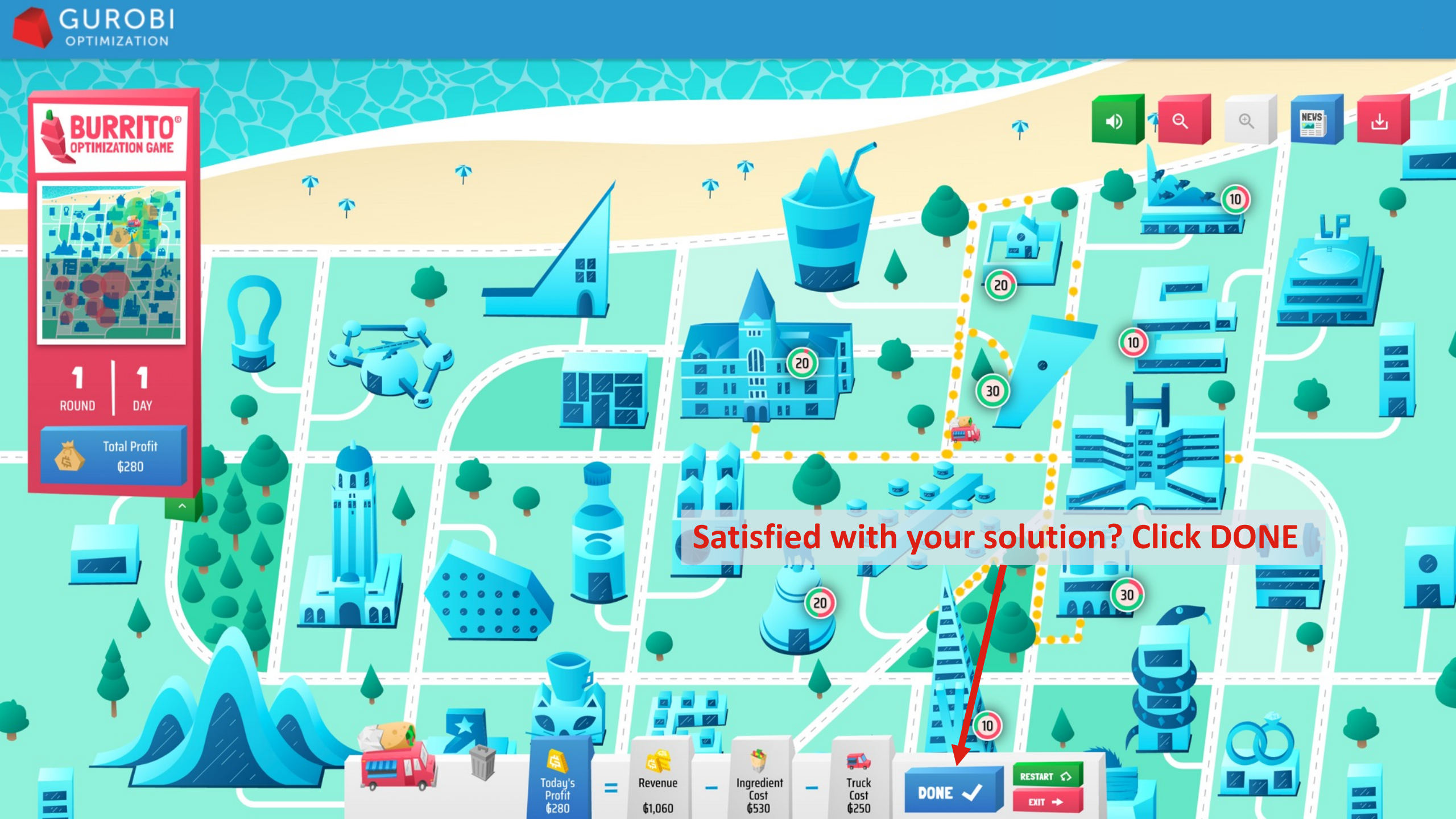

GUROBI
OPTIMIZATION
BURRITO®
OPTIMIZATION GAME
1
ROUND
1
DAY
Total Profit
₲0
The “newsfeed”
TODAY
ROUND 1 DAY 1
Grand opening of Guroble burrito trucks today! How many Gurobucks (₲) profit can you earn?
Sales revenue ₲10 per burrito sold
Ingredient cost ₲5 per burrito sold
Truck cost ₲250 per truck opened
OK
Pay attention to today’s costs and revenue
DRAG TRUCK TO MAP TO START
DRAG TRUCKS TO BIN OR CLICK TO CLEAR ALL
Today's Profit ₲0
=
Revenue ₲0
–
Ingredient Cost ₲0
–
Truck Cost ₲0
DONE
RESTART
EXIT
NEWS
LP
10
20
30


GUROBI
OPTIMIZATION
BURRITO®
OPTIMIZATION GAME
1
ROUND
1
DAY
Total Profit
₲280
NEWS
Your costs, revenues, and profit for today
Today's Profit
₲280
=
Revenue
₲1,060
−
Ingredient Cost
₲530
−
Truck Cost
₲250
DONE
RESTART
EXIT


GUROBI
OPTIMIZATION
BURRITO®
OPTIMIZATION GAME
1
ROUND
1
DAY
Total Profit
₲280
NEWS
10
20
30
20
10
30
20
10
There’s more map this way! (Scroll or zoom to see it)
Today's Profit
₲280
=
Revenue
₲1,060
−
Ingredient Cost
₲530
−
Truck Cost
₲250
DONE
RESTART
EXIT


30
Maximum number of customers at this building today
Fraction you haven't
30
20
Fraction you've "captured"
(Capture more by placing trucks closer to buildings)
10


GUROBI
OPTIMIZATION
BURRITO®
OPTIMIZATION GAME
1
ROUND
1
DAY
Total Profit
₲280
NEWS
Drag truck to new spot to move it
Or to trash to remove it
Today's Profit
₲280
=
Revenue
₲1,060
−
Ingredient Cost
₲530
−
Truck Cost
₲250
DONE
RESTART
EXIT


GUROBI
OPTIMIZATION
BURRITO®
OPTIMIZATION GAME
1
ROUND
1
DAY
Total Profit
₲280
NEWS
Satisfied with your solution? Click DONE
Today's Profit ₲280
=
Revenue ₲1,060
−
Ingredient Cost ₲530
−
Truck Cost ₲250
DONE
RESTART
EXIT

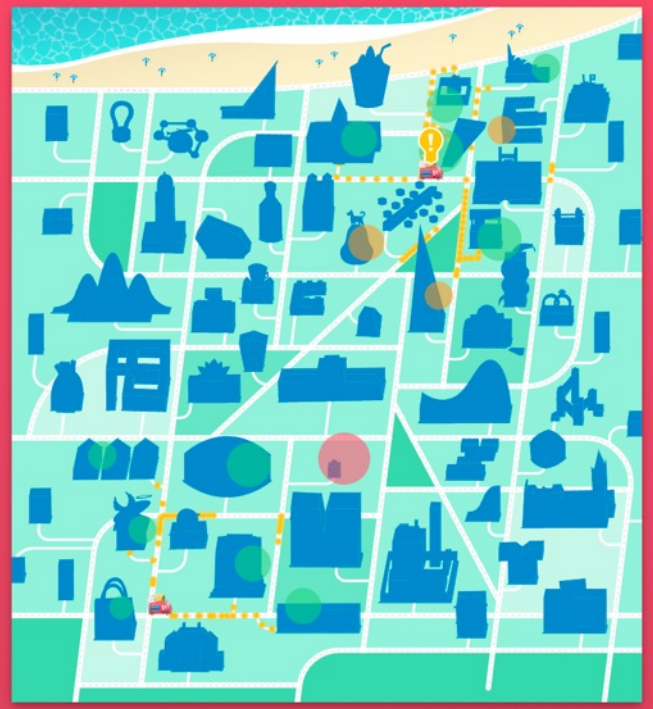
GUROBI OPTIMIZATION

BURRITO® OPTIMIZATION GAME

1 ROUND | 1 DAY

Total Profit ₲400

YOUR SOLUTION



Sales revenue: ₲1,800
Ingredient cost: ₲900
Truck cost: ₲500
TOTAL PROFIT: ₲400 (33% worse than optimal)

OPTIMAL SOLUTION

Sales revenue: ₲2,200
Ingredient cost: ₲1,100
Truck cost: ₲500
TOTAL PROFIT: ₲600

Gurobi found an optimal solution in **0.242 seconds**

LEGEND

- Truck location
- Fewer customers
- More customers
- <25% of demand captured
- 25%-50% of demand captured
- >50% of demand captured
- Tip

TRY AGAIN

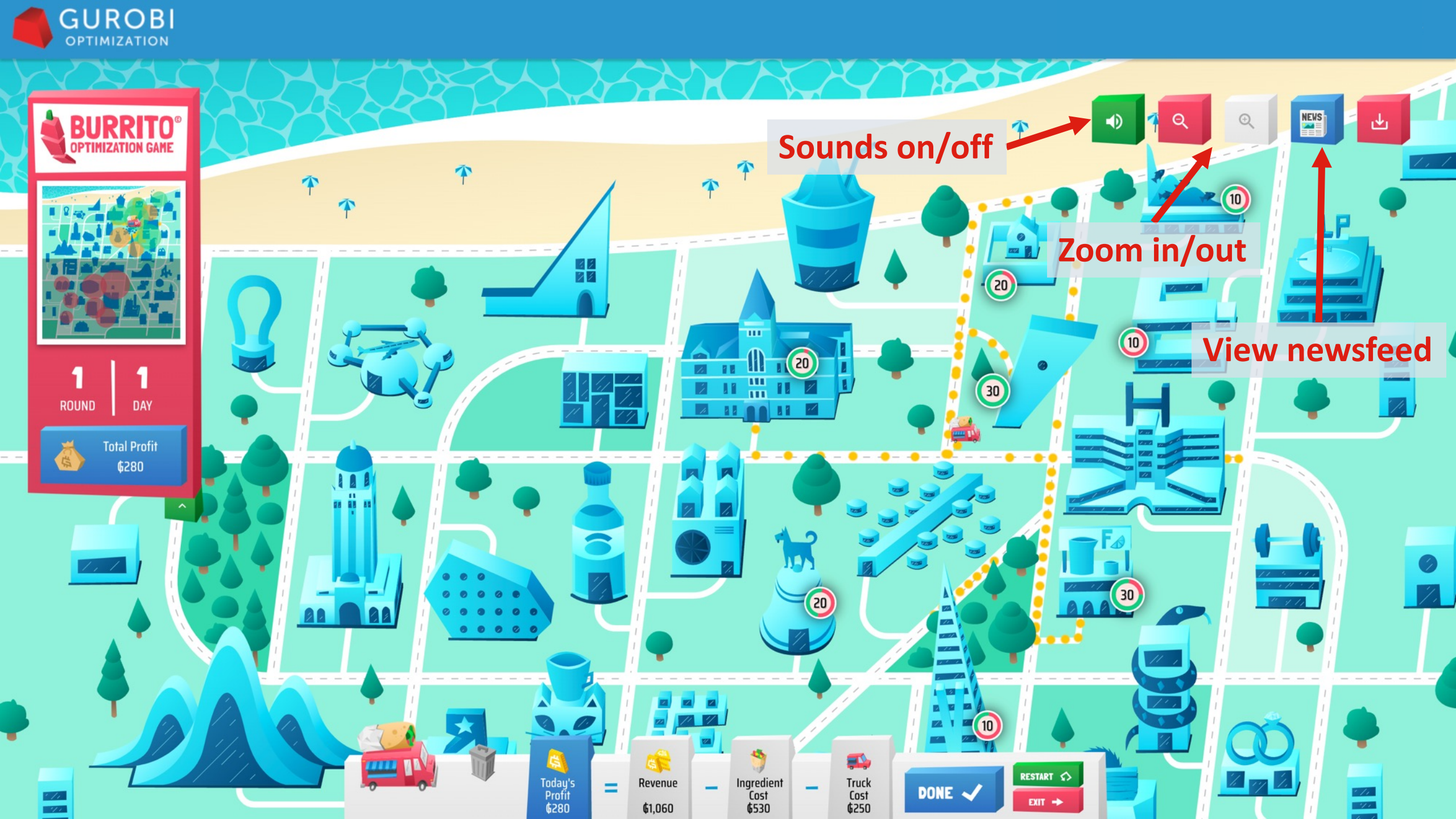
NEXT DAY

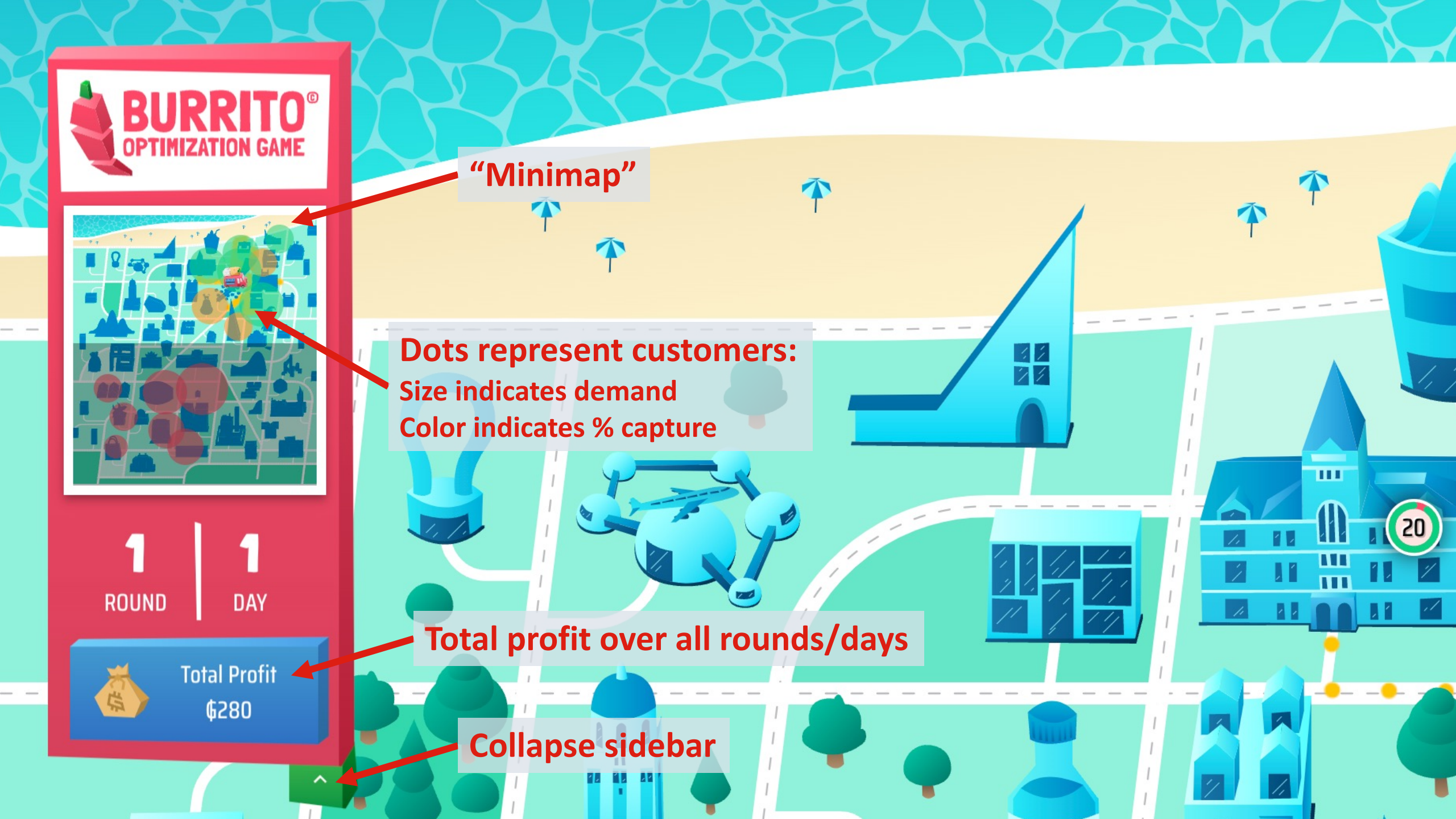
Today's Profit ₲400 = Revenue ₲1,800 − Ingredient Cost ₲900 − Truck Cost ₲500

DONE

RESTART

EXIT


GUROBI
OPTIMIZATION
BURRITO®
OPTIMIZATION GAME
1
ROUND
1
DAY
Total Profit
¢280
Sounds on/off
Zoom in/out
View newsfeed
NEWS
Today's Profit ¢280
=
Revenue ¢1,060
−
Ingredient Cost ¢530
−
Truck Cost ¢250
DONE
RESTART
EXIT


BURRITO©
OPTIMIZATION GAME
1
ROUND
1
DAY
Total Profit
₲280
20
"Minimap"
Dots represent customers:
Size indicates demand
Color indicates % capture
Total profit over all rounds/days
Collapse sidebar

GUROBI OPTIMIZATION

Coping with Uncertainty

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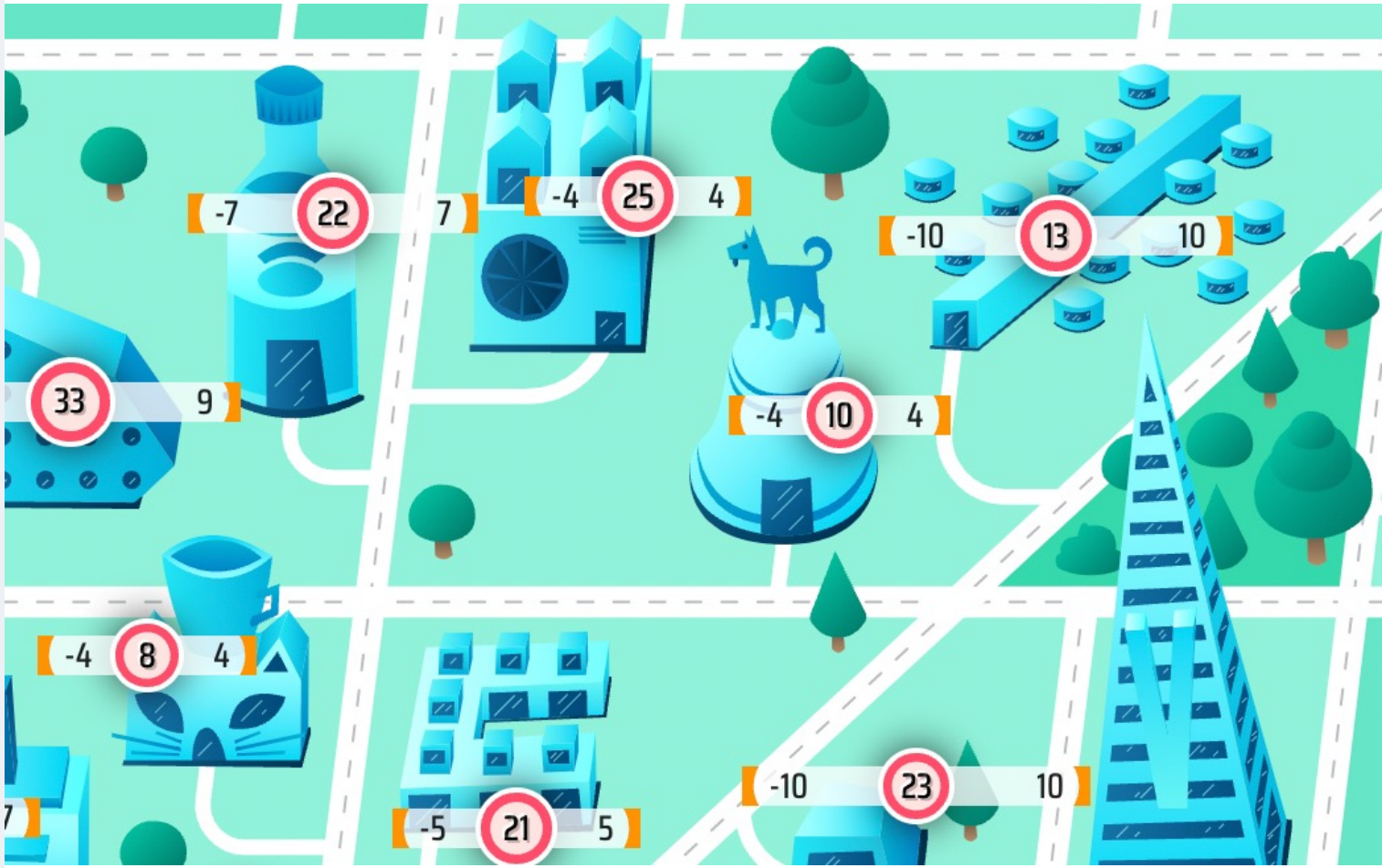
Round 2

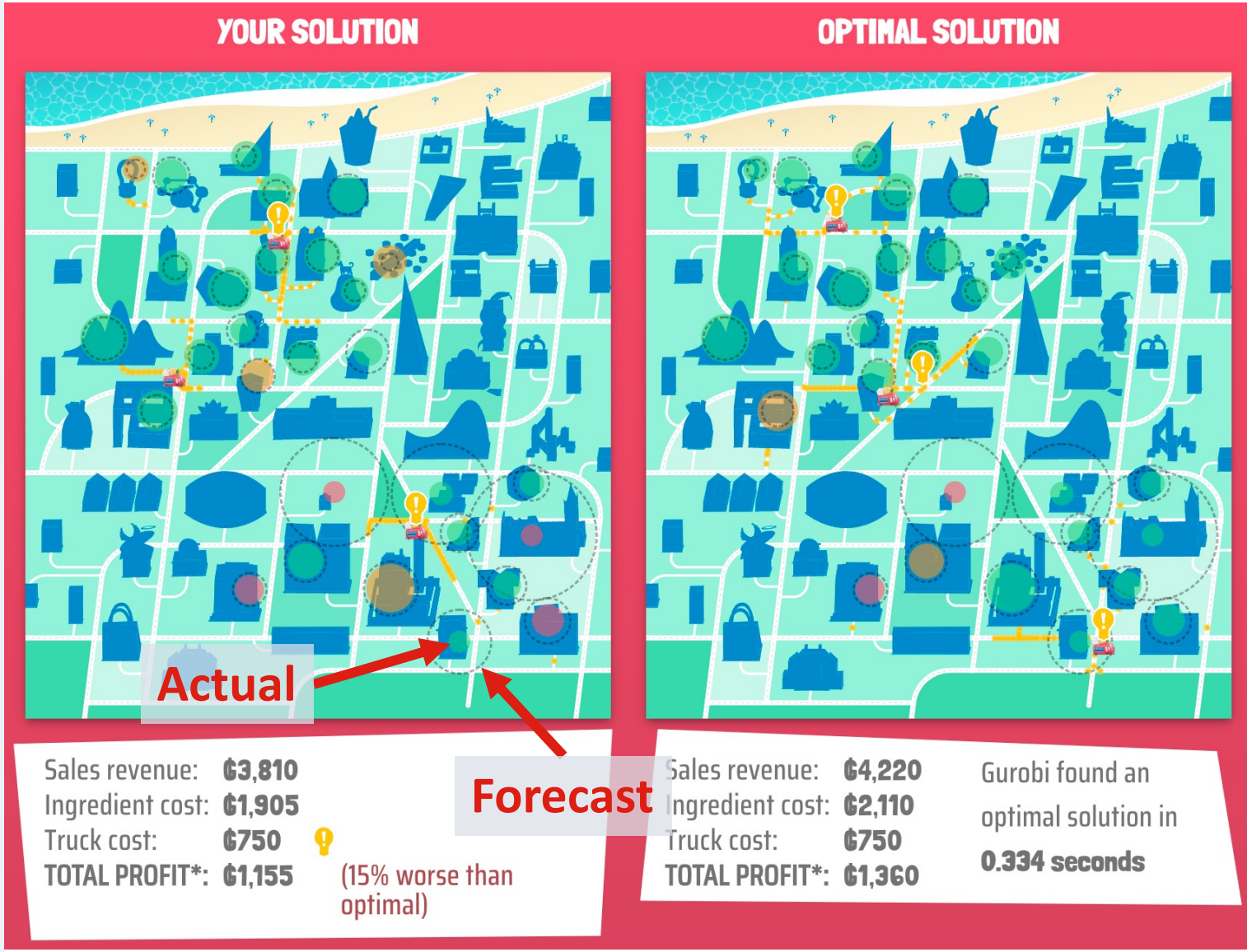
- The numbers on the buildings represent demand forecasts.
- Error bars indicate the range of possible values.



GUROBI OPTIMIZATION

Round 2

- Your solution (and Gurobi's) are **evaluated** based on actual demands
- Even though you (and Gurobi) only have forecasts when **planning**.



How should we optimize when demands are random?

GUROBI OPTIMIZATION

maximize $\Pi(\widehat{D})$	maximize $\Pi(\mathbb{E}_D[D])$	maximize $\mathbb{E}_D[\Pi(D)]$	maximize $\min_D[\Pi(D)]$

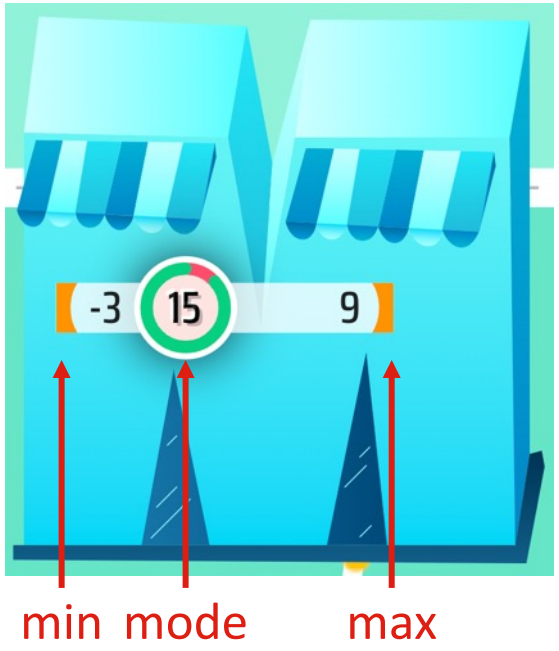
$\Pi(D)$=profit if demands=D

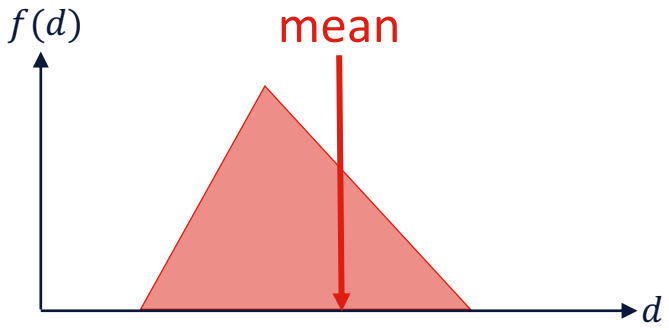
Optimize **performance under the forecasts**	Optimize **performance under the means**	Optimize the **mean performance**	Optimize the **worst-case performance**

These are equivalent, given the structure of the Burrito optimization problem.
This is how Gurobi optimizes in the game.

Into the Weeds

The random demands follow a triangular distribution:

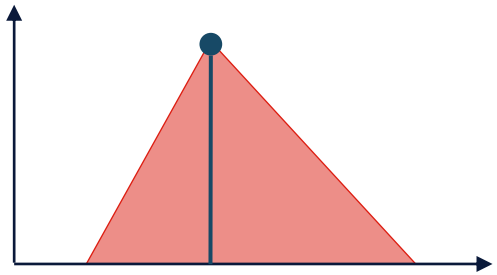


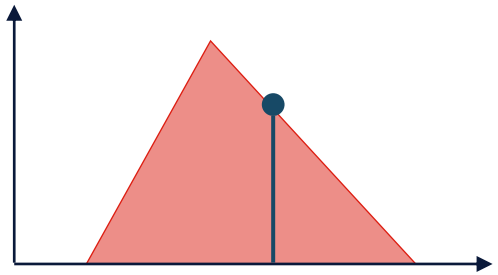


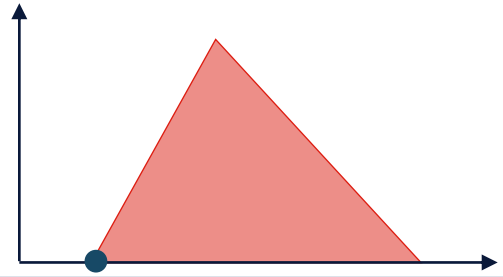
How should we optimize when demands are random?

GUROBI OPTIMIZATION

maximize $\Pi(\widehat{D})$	maximize $\Pi(\mathbb{E}_D[D])$	maximize $\mathbb{E}_D[\Pi(D)]$	maximize $\min_D[\Pi(D)]$
Optimize **performance under the forecasts**	Optimize **performance under the means**	Optimize the **mean performance**	Optimize the **worst-case performance**







Optimization terminology

maximize $\Pi(\widehat{D})$	maximize $\Pi(\mathbb{E}_D[D])$	maximize $\mathbb{E}_D[\Pi(D)]$	maximize $\min_D[\Pi(D)]$
Optimize **performance under the forecasts**	Optimize **performance under the means**	Optimize the **mean performance**	Optimize the **worst-case performance**
Deterministic Optimization		Stochastic Optimization	Robust Optimization

GUROBI OPTIMIZATION

Optimization 101

Mathematical Optimization

GUROBI OPTIMIZATION

In mathematical optimization, the goal is to

minimize/maximize	some performance metric	"objective function"
by choosing	some quantities	"decision variables"
subject to	some restrictions on our decisions	"constraints"

For example:

minimize	manufacturing cost
by choosing	production quantities at each factory
subject to	produce enough to meet demand

Mathematical Optimization

GUROBI OPTIMIZATION

In mathematical optimization, the goal is to

minimize/maximize	some performance metric
by choosing	some quantities
subject to	some restrictions on our decisions

For example:

maximize	operating room (OR) utilization
by choosing	sequence of surgeries in each OR
subject to	higher-priority patients scheduled first setup time between surgeries

Mathematical Optimization

GUROBI OPTIMIZATION

In mathematical optimization, the goal is to

minimize/maximize	some performance metric
by choosing	some quantities
subject to	some restrictions on our decisions

For example:

maximize	profit
by choosing	burrito truck locations
subject to	each customer visits ≤1 truck

Formulating Problems Mathematically

GUROBI OPTIMIZATION

We can't write these quantities in words. We need to write them mathematically.

Here's the Burrito Optimization Game problem, formulated mathematically:

$$\text{maximize} \quad \sum_{i \in I} \sum_{j \in J} (r - k) \alpha_{ij} d_i \, y_{ij} - \sum_{j \in J} f_j x_j$$

$$\text{subject to} \quad \sum_{j \in J} y_{ij} \leq 1 \qquad \forall i \in I$$

$$y_{ij} \leq x_j \qquad \forall i \in I, j \in J$$

$$x_j, y_{ij} \in \{0,1\} \quad \forall i \in I, j \in J$$

(You don't need to understand this formulation!)

Classifying Mathematical Optimization Problems

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Mathematical optimization problems are classified using terms like

- linear programming (LP)
- integer programming (IP)
- mixed-integer programming (MIP)
- nonlinear programming (NLP)
- constraint programming (CP)
- stochastic programming (SP)

based on the types of objective function, decision variables, and constraints.

Solving Mathematical Optimization Problems

GUROBI OPTIMIZATION

Once you have formulated an optimization problem, how do you solve it?

That is, how do you find the optimal solution?

Some possible approaches:

By hand/trial-and-error

This is essentially what you did when you played the Burrito Optimization Game.

It might be fun (for a while), but it is time-consuming and usually does not find optimal solutions.

Try all solutions

Computers are *fast*. Why not program the computer to check every possible solution? Then just pick the best one.

This approach is called total enumeration. But it's a bad idea. Let's see why…

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Don't Use Enumeration!

Suppose the computer can evaluate

1,000,000,000

solutions per second.

# Nodes	# Solutions	Time	What You Could Do in that Time
10	1,024	0.000001 sec	nothing
20	1,048,576	0.001 sec	start to blink
30	1,073,741,824	1.1 sec	smile
35	34,359,738,368	34.4 sec	listen to 1/3 of "Baby Shark"
40	1 trillion	18.3 min	watch most of one episode of *The Office*
45	35 trillion	9.8 hours	watch first 2.5 *Lord of the Rings* movies
50	1 quadrillion	13.0 days	watch every episode of *The Office* 4.2x
55	36 quadrillion	1.1 years	get a Master's degree
60	1 quintillion	36.6 years	pay off your mortgage
70	1 sextillion	37,436 years	travel to Alpha Centauri and back 3x
80	1 septillion	38 million years	almost travel to the nearest black hole

Solving Mathematical Optimization Problems

GUROBI OPTIMIZATION

Some other approaches:

Common-sense rules of thumb	Divide the problem (e.g., the Burritoville map) into smaller problems and solve those.
	Make decisions (e.g., truck locations) one at a time.
	Simplify the problem (e.g., ignore demand numbers).

Approximate approaches like these are called heuristics.

They are useful and important in practice.

Solving Mathematical Optimization Problems

Other approaches:

Optimization “solvers”

These software packages use math and algorithms to find optimal solutions. Gurobi offers a free license to all academics along with resources to help you get started. To download your license, please visit: gurobi.com/academic

GUROBI OPTIMIZATION

www.BurritoOptimizationGame.com